ALGEBRA ESERCIZI

martedì 2 novembre 2021 21:15

Si consideri il sottogruppo H di Z4 generato dagli elementi (1, 2, 2, 4), (0, 4, 8, 2), (1, 2, 8, 0). Descrivere Z4/H

compitino.

$$H = \mathrm{Span}\left(\begin{pmatrix} 1 \\ 2 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 8 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 8 \\ 0 \end{pmatrix} \right)$$

$\mathbb{Z}^4 / H$ cerco $G \cong H$

Come cerco G?

$$\begin{pmatrix} 1 & 0 & 1 \\ 2 & 4 & 2 \\ 2 & 8 & 8 \\ 4 & 2 & 0 \end{pmatrix}$$

voglio 1 solo coeff non nullo in ogni colonna.

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 4 & 0 \\ 0 & 8 & 6 \\ 0 & 2 & -4 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -4 \\ 0 & 8 & 6 \\ 0 & 4 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbb{Z}^4 / H \cong \mathbb{Z}^4 / \mathrm{Span}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \\ 0 \end{pmatrix} \right)$$

$$\mathbb{Z}^4 / H \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}$$

Sia = (1, 2, 3, 4, 5) 2 A5. Quanti elementi ha l'orbita di rispetto all'azione di A5 su se stesso per coniugio?[

$$\theta = (1\,2\,3\,4\,5) \in A_5$$

$$A_5 \circlearrowright A_5 \qquad A_5 \times A_5 \to A_5$$

$$(\tau, \gamma) \to \tau \gamma \tau^{-1}$$

$|orb(\sigma)|$? $\dfrac{|A5|}{|Stab(\sigma)|}$ $\quad Stab\,\sigma < S5$

$\|$ $\quad$ $|$

$gxg^{-1} = x$ $\quad$ $< A5$ V metà p. d.

$Stab\,\sigma < A5 \iff$ solo cicli di ordine pari

Solo cicli di ordine pari di lung. diversa.

$|Stab| = 5$. $\quad orb\,\sigma = \dfrac{60}{5} = 12$

$X = \{$ sgr. di ordine 5 in $A5 \}$

$|X|$ $\quad$ $4! = \dfrac{24}{4}$ = # elt. di ordine

$6 = |X|$

$\langle\sigma\rangle$ $\quad |\langle\sigma\rangle| = 5$

$A5$

$|G| = 66 = 2\cdot 3\cdot 11 = p\cdot q\cdot r$

N_{11}

$n_{11} \mid 6$

$n_{11} \equiv 1\ (11) \Rightarrow N_{11} \triangleleft G$

N_3

$n_3 \mid 22$ $\quad n_3 \to 1,\ 22$

$n_3 \equiv 1\ (3)$

$1 + 2\cdot 22 = 45 + 10 = 55 \quad (11)$

N_2

$n_2 \mid 33$ $\quad n_2 = 1,\ 3,\ 11,\ \cancel{33}$

$\equiv 1\ (2)$

$3 \nmid 11 - 1 \Rightarrow \mathbb{Z}_3 \triangleleft G.$

Se $2 \mid |G| \wedge 4 \nmid |G| \Rightarrow \exists\, H < G$ t.c. $|G/H| = 2.$

Quindi $2 \mid 66$ ma $4 \nmid 66 \Rightarrow \exists$ in G $H < G$ t.c.

$|G/H| = 2 \Rightarrow H \cong \mathbb{Z}_{33} \cong \mathbb{Z}_3 \times \mathbb{Z}_{11} \Rightarrow n_3 = 1 \Rightarrow N_3 \triangleleft G.$

$\boxed{\mathbb{Z}_{11} \rtimes \mathbb{Z}_3}$

$N_3 \triangleleft H \quad n_3 \equiv 1\ (3)$ e $n_3 \mid 11 \Rightarrow n_3 = 1.$

$N_3 \cdot N_{11} \triangleleft G \qquad \mathbb{Z}_{33}$

$G = \mathbb{Z}_{33} \rtimes_{\varphi} \mathbb{Z}_2$

$\varphi : \mathbb{Z}_2 \longrightarrow Aut(\mathbb{Z}_{33}) = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_5 .$

$\varphi_{ban} \quad 1 \longrightarrow Id \longrightarrow 0$

$1 \longrightarrow \qquad \longrightarrow (1, 0, 0) \longrightarrow$

$(0, 1, 0)$

$(1, 1, 0)$

$\mathbb{Z}_2 \times \mathbb{Z}_{10}$

· $\mathbb{Z}_{33} \times \mathbb{Z}_2 \cong \mathbb{Z}_{66}$

$\underline{\alpha \in Aut(\mathbb{Z}_{33})}$

$\alpha \, \tau_1(1) \, \alpha^{-1} = \tau_2(1)$

$\mathbb{Z}_3^* \times \mathbb{Z}_{11}^*$

1	1
2	1
1	10
2	10

$(\mathbb{Z}_3 \rtimes \mathbb{Z}_2) \times \mathbb{Z}_{11}$

$\underline{D_3 \times \mathbb{Z}_{11}}$

$(\mathbb{Z}_{11} \rtimes \mathbb{Z}_2) \times \mathbb{Z}_3$

$\underline{D_{11} \times \mathbb{Z}_3}$

i $\mathbb{Z}_3$

$D_{2 3}$

Se il ce $\left(\mathbb{Z}_p \times \mathbb{Z}_q\right.$