

Esercitazione 10

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COMPOSIZIONE DI APP. LINEARI

$f: V \rightarrow W$ $g: W \rightarrow Z$ lineari $\Rightarrow g \circ f$ lineare

$$\text{Ker } f \subseteq \text{Ker } g \circ f$$

$$(g \circ f)(v) = g(f(v)) = g(0) = 0 \quad v \in \text{Ker } f.$$

$$\text{Im } g \supseteq \text{Im } g \circ f$$

$$\exists v \in V \text{ t.c. } z = g(f(v)) \in \text{Im } g.$$

$$\text{rk}(g \circ f) = \dim \text{Im}(g \circ f) \leq \dim \text{Im}(g) = \text{rk } g$$

$$\begin{aligned} \text{rk}(g \circ f) &= \dim V - \dim \text{Ker}(g \circ f) \leq \dim V - \dim \text{Ker } f = \\ &= \dim \text{Im } f = \text{rk } f \end{aligned}$$

$$\dim \text{Ker } g \circ f = \dim V - \text{rk}(g \circ f) = \underbrace{\dim V - \text{rk } f}_{\dim \text{Ker } f} + \dim(\text{Im } f \cap \text{Ker } g) + \dim \text{Im } f \cap \text{Ker } g.$$