

Esercitazione 16

venerdì 7 gennaio 2022 11:54

Calcolo \det con la place

$$A = \begin{pmatrix} -1 & 2 & 3 \\ -2 & 0 & 1 \\ 1 & 0 & -2 \end{pmatrix} \quad \begin{array}{c} + - + \\ - + - \\ + - + \end{array}$$

$$\det A = -1 \det \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix} + 2 \det \begin{pmatrix} 2 & 3 \\ 0 & -2 \end{pmatrix} + 1 \det \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} = \\ = -1 \cdot 0 + 2(-4) + 1 \cdot 2 = -8 + 2 = -6$$

$$\det A = 2 \det \begin{pmatrix} 2 & 3 \\ 0 & -2 \end{pmatrix} + 0 \det \begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix} - 1 \det \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix} = -6$$

$$\det A = -2 \det \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} + 2 + 0 = -6.$$

A. G e \det

B è ottenuto da A

- op I tipo $\Rightarrow \det B = -\det A$
- op II tipo $\Rightarrow \det B = \lambda \det A$
- op III tipo $\Rightarrow \det B = \det A$

$$\det \begin{pmatrix} 1 & 0 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & -2 & -1 & 0 \\ -1 & 3 & -2 & 2 \end{pmatrix} \quad \text{Tramite la place}$$

$$(-1) \det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ -1 & 3 & -2 \end{pmatrix} + 2 \det \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 1 & -2 & -1 \end{pmatrix} = \\ = (-1) \left[2 \det \begin{pmatrix} -2 & -1 \\ 3 & -2 \end{pmatrix} - 1 \det \begin{pmatrix} 1 & 1 \\ 3 & -2 \end{pmatrix} - 1 \det \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} \right] +$$

$$+2 \left[1 \det \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} + 2 \det \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right] =$$

$$-1 [2(4+3) - 1(-2-3) - 1(-1+2)] +$$

$$+2[1(-1-2) + 2(1-6)] =$$

$$= -1[14+5-1] + 2[-3-6] = -18-18 = -36.$$

Tramite Gauss

$$\left| \begin{array}{cccc} 1 & 0 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & -2 & -1 & 0 \\ -1 & 3 & -2 & 2 \end{array} \right| \xrightarrow{R_4 \leftarrow R_4 - 2R_1} \det \left| \begin{array}{cccc} 1 & 0 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & -2 & -1 & 0 \\ -3 & 3 & -6 & 0 \end{array} \right| =$$

$$= -1 \det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ -3 & 3 & -6 \end{pmatrix} = 3 \det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & -1 & 2 \end{pmatrix} =$$

$$3 \det \begin{pmatrix} 0 & 3 & -3 \\ 0 & -1 & -3 \\ 1 & -1 & 2 \end{pmatrix} = 3 \cdot \det \begin{pmatrix} 3 & -3 \\ -1 & -3 \end{pmatrix} = (-9-3)3 \\ = -36.$$