

## Esercitazione 5

lunedì 3 gennaio 2022 16:30

### SOMMA DI SSP

$$U_1, U_2 \subset V$$

$$W = \{ Z \mid$$

$$Z \subset V$$

$$Z \supset U_1 \cup U_2$$

$$\bullet W = \text{Span}(U_1 \cup U_2)$$

dim

$$\subset \text{Span}(U_1 \cup U_2) \supset U_1 \cup U_2 \Rightarrow \text{Span}(U_1 \cup U_2) \text{ è uno degli } Z$$

$$\Rightarrow W \subseteq \text{Span}(U_1 \cup U_2)$$

$$\supseteq \underbrace{U_1 \cup U_2} \subseteq W$$

$$\stackrel{||}{\text{Span}}(U_1 \cup U_2) \Rightarrow \text{Span}(U_1 \cup U_2) \subseteq W.$$

$$\bullet S, T \text{ sottoinsiemi di } V$$

$$\text{Span}(S) + \text{Span}(T) = \text{Span}(S \cup T)$$

dim

$\subset$

$$S \subseteq S \cup T \Rightarrow \text{Span}(S) \subseteq \text{Span}(S \cup T)$$

$$T \subseteq S \cup T \Rightarrow \text{Span}(T) \subseteq \text{Span}(S \cup T)$$

$$\Rightarrow \text{Span}(S) + \text{Span}(T) \subseteq \text{Span}(S \cup T)$$

$$\supseteq \text{Span}(S \cup T) \subset \text{Span}(S) + \text{Span}(T)$$

$$S \subseteq \text{Span}(S), \quad T \subseteq \text{Span}(T)$$

$$\left. \begin{array}{l} \exists s \in S \quad \exists = 0 + s \in \text{Span}(S) \\ t \in T \quad t = 0 + t \in \text{Span}(T) \end{array} \right\}$$

$$s+t \in \text{Span}(S) + \text{Span}(T)$$

## • esercizio

$$f: \mathbb{R}^2 \rightarrow M(2, \mathbb{R})$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x-y & x+2y \\ 3x+y & x+y \end{pmatrix}$$

1) Verificare che  $\bar{e}$  lineare  $\rightarrow$  facile.

2) Trovare ker e Im

3) Dire se  $\bar{e}$  bigettiva

$\hookrightarrow$

$$2) \text{ Ker } f = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid \begin{pmatrix} x-y & x+2y \\ 3x+y & x+y \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \right\}$$

$$\begin{cases} x-y=0 & \rightarrow x=y \\ x+2y=0 \end{cases}$$

$$\begin{cases} 3x+y=0 \\ x+y=0 & \rightarrow x=-y \end{cases}$$

$$\Rightarrow x=y=0 \Rightarrow f \bar{e} \text{ inj.}$$

$$\Rightarrow x=y=0 \Rightarrow f \bar{e} \text{ inj.}$$

surj.

$$\begin{pmatrix} x-y & x+2y \\ 3x+y & x+y \end{pmatrix} = \begin{pmatrix} x & x \\ 3x & x \end{pmatrix} + \begin{pmatrix} -y & 2y \\ y & y \end{pmatrix} =$$

$$= x \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix} + y \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}$$

$$\text{Im } f = \text{Span} \left( \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix} \right)$$

$\text{Im } f \subsetneq M(2, \mathbb{R})$  es.  $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$  non è comb lineare.  
a e b.