# MLD2P4 User's and Reference Guide

A guide for the Multi-Level Domain Decomposition Parallel Preconditioners Package based on PSBLAS

> Pasqua D'Ambra ICAR-CNR, Naples, Italy

Daniela di Serafino Second University of Naples, Italy

Salvatore Filippone University of Rome "Tor Vergata", Italy

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#### Abstract

MLD2P4 (MULTI-LEVEL DOMAIN DECOMPOSITION PARALLEL PRE-CONDITIONERS PACKAGE BASED ON PSBLAS) is a package of parallel algebraic multi-level preconditioners. It implements various versions of one-level additive and of multi-level additive and hybrid Schwarz algorithms. In the multi-level case, a purely algebraic approach is applied to generate coarse-level corrections, so that no geometric background is needed concerning the matrix to be preconditioned. The matrix is required to be square, real or complex, with a symmetric sparsity pattern

MLD2P4 has been designed to provide scalable and easy-to-use preconditioners in the context of the PSBLAS (Parallel Sparse Basic Linear Algebra Subprograms) computational framework and can be used in conjuction with the Krylov solvers available in this framework. MLD2P4 enables the user to easily specify different aspects of a generic algebraic multilevel Schwarz preconditioner, thus allowing to search for the "best" preconditioner for the problem at hand. The package has been designed employing object-oriented techniques, using Fortran 95 and MPI, with interfaces to additional external libraries such as UMFPACK, SuperLU and SuperLU\_Dist, that can be exploited in building multi-level preconditioners.

This guide provides a brief description of the functionalities and the user interface of MLD2P4.

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### 1 General Overview

The MULTI-LEVEL DOMAIN DECOMPOSITION PARALLEL PRECONDITIONERS PACKAGE BASED ON PSBLAS (MLD2P4) provides *multi-level Schwarz pre-conditioners* [?], to be used in the iterative solutions of sparse linear systems:

$$Ax = b, \tag{1}$$

where A is a square, real or complex, sparse matrix with a symmetric sparsity pattern. These preconditioners have the following general features:

- both *additive and hybrid multilevel* variants, i.e. multiplicative among the levels and additive inside a level, are implemented; the basic additive Schwarz preconditioners are obtained by considering only one level;
- a *purely algebraic* approach is used to generate a sequence of coarse-level corrections to a basic preconditioner, without explicitly using any information on the geometry of the original problem (e.g. the discretization of a PDE). The *smoothed aggregation* technique is applied as algebraic coarsening strategy [?, ?].

The package is written in *Fortran 95*, following an *object-oriented approach* through the exploitation of features such as abstract data type creation, functional overloading and dynamic memory management, while providing a smooth path towards the integration in legacy application codes. The parallel implementation is based on a Single Program Multiple Data (SPMD) paradigm for distributed-memory architectures. Single and double precision implementations of MLD2P4 are available for both the real and the complex case, that can be used through a single interface. **SALVATORE**, funziona tutto?

MLD2P4 has been designed to implement scalable and easy-to-use multilevel preconditioners in the context of the PSBLAS (Parallel Sparse BLAS) computational framework [10]. PSBLAS is a library originally developed to address the parallel implementation of iterative solvers for sparse linear system, by providing basic linear algebra operators and data management facilities for distributed sparse matrices; it also includes parallel Krylov solvers, built on the top of the basic PSBLAS kernels. The preconditioners available in MLD2P4 can be used with these Krylov solvers. The choice of PSBLAS has been mainly motivated by the need of having a portable and efficient software infrastructure implementing "de facto" standard parallel sparse linear algebra kernels, to pursue goals such as performance, portability, modularity ed extensibility in the development of the preconditioner package. On the other hand, the implementation of MLD2P4 has led to some revisions and extentions of the PSBLAS kernels, leading to the recent PSBLAS 2.0 version [?]. The inter-process comunication required by MLD2P4 is encapsulated into the PSBLAS routines, except few cases where MPI [17] is explicitly called. Therefore, MLD2P4 can be run on any parallel machine where PSBLAS and MPI implementations are available.

MLD2P4 has a layered and modular software architecture where three main layers can be identified. The lower layer consists of the PSBLAS kernels, the middle one implements the construction and application phases of the preconditioners, and the upper one provides a uniform and easy-to-use interface to all the preconditioners. This architecture allows for different levels of use of the package: few black-box routines at the upper layer allow non-expert users to easily build any preconditioner available in MLD2P4 and to apply it within a PSBLAS Krylov solver. On the other hand, the routines of the middle and lower layer can be used and extended by expert users to build new versions of multi-level Schwarz preconditioners. We provide here a description of the upper-layer routines, but not of the medium-layer ones.

This guide is organized as follows:organizzazione della guida

# 2 Notational Conventions

- caratteri tipografici usati nella guida (vedi guida ML recente e guida Aztec)

- convenzioni sui nomi di routine (differenza nei nomi tra high-level e medium-

level), strutture dati, moduli, costanti, etc. (vedi guida p<br/>sblas)  $\,$ 

- versione reale e complessa, singola e doppia precisione

# 3 Code Distribution

The MLD2P4 is freely distributable under the following copyright terms:

MLD2P4 version 1.0 MultiLevel Domain Decomposition Parallel Preconditioners Package based on PSBLAS (Parallel Sparse BLAS version 2.3)

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Salvatore Filippone	University of Rome Tor Vergata					
Alfredo Buttari	University of Rome Tor Vergata					
Pasqua D'Ambra	ICAR-CNR, Naples					
Daniela di Serafino	Second University of Naples					

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# 4 Configuring and Building MLD2P4

- uso di GNU autoconf e automake

- software di base necessario (MPI, BLACS, BLAS, PSBLAS, UMFPACK ? - specificare versioni)

- software opzionale (SuperLU, SuperLUdist - specificare versioni e opzioni di configure)

- sistemi operativi e compilatori su cui MLD2P4 e' stato costruito con successo

- sono previste opzioni di configurazione per il debugging o per il profiling?

- albero delle directory generato al momento dell'installazione

# 5 Getting Started

We describe the basics for building and applying MLD2P4 one-level and multilevel Schwarz preconditioners with the Krylov solvers included in PSBLAS []. The following steps are required:

- Declare the preconditioner data structure. It is a derived data type, mld\_xprec\_type, where x may be s, d, c or z, according to the basic data type of the sparse matrix (s = real single precision; d = real double precision; c = complex single precision; z = complex double precision). This data structure is accessed by the user only through the MLD2P4 routines, following an object-oriented approach.
- 2. Allocate and initialize the preconditioner data structure, according to a preconditioner type chosen by the user. This is performed by the routine mld\_precinit, which also sets defaults for each preconditioner type selected by the user. The defaults associated to each preconditioner type are listed in Table 1, where the strings used by mld\_precinit to identify the preconditioner types are also given.
- 3. Modify the selected preconditioner type, by properly setting preconditioner parameters. This is performed by the routine mld\_precset. This routine must be called only if the user wants to modify the default values of the parameters associated to the selected preconditioner type, to obtain a variant of the preconditioner. Examples of use of mld\_precset is given in Section 5.1; a complete list of all the preconditioner parameters and their allowed and default values is provided in Section 6, Tables 2-5.
- 4. Build the preconditioner for a given matrix. This is performed by the routine mld\_precbld.
- 5. Apply the preconditioner at each iteration of a Krylov solver. This is performed by the routine mld\_precaply. When using the PSBLAS Krylov solvers, this step is completely transparent to the user, since mld\_precaply is called by the PSBLAS routine implementing the Krylov solver (psb\_krylov).
- 6. Free the preconditioner data structure. This is performed by the routine mld\_precfree. This step is complementary to step 1 and should be performed when the preconditioner is no more used.

A detailed description of the above routines is given in Section 6.

Note that the Fortran 95 module mld\_prec\_mod must be used in the program calling the MLD2P4 routines. Furthermore, to apply MLD2P4 with the Krylov solvers from PSBLAS, the module psb\_krylov\_mod must be used too. DOBBIAMO SPECIFICARE QUALCHE ALTRO MODULO, AD ESEMPIO psb\_base\_mod? Examples showing the basic use of MLD2P4 are reported in Section 5.1.

**Remark.** The coarsest-level solver used by the default two-level preconditioner has been chosen by taking into account that, on parallel machines, it often leads to the smallest execution time when applied to linear systems coming from finitedifference discretizations of basic elliptic PDE problems, considered as standard tests for multi-level Schwarz preconditioners [3, 2]. However, this solver does not

#### <u>1 General Overview</u>

correspond to the smallest number of iterations of the preconditioned Krylov method, which is usually obtained by applying a direct solver, e.g. based on the LU factorization, at the coarsest level (see Section 6 for coarsest-level solvers available in MLD2P4).

Type	String	Default preconditioner
No preconditioner	'NOPREC'	(Considered only to use the PSBLAS Krylov
		solvers with no preconditioner.)
Diagonal	'DIAG'	
Block Jacobi	'BJAC'	Block Jacobi with $ILU(0)$ on the local
		blocks.
Additive Schwarz	'AS'	Restricted Additive Schwarz (RAS), with
		overlap 1 and $ILU(0)$ on the local blocks.
Multilevel	'ML'	Multi-level hybrid preconditioner (additive
		on the same level and multiplicative through
		the levels), with post-smoothing only. Num-
		ber of levels: 2; post-smoother: RAS with
		overlap 1 and with $ILU(0)$ on the local
		blocks; coarsest matrix: distributed among
		the processors; (approximate) coarse-level
		solver: 4 sweeps of the block-Jacobi solver,
		with the UMFPACK LU factorization on
		the blocks (double precision versions) or
		XXXXXXXXXXX (single precision versions)

Table 1: Preconditioner types, corresponding strings and default choices.

#### 5.1 Examples

The code reported in Figure 1 shows how to set and apply the default multilevel preconditioner available in the real double precision version of MLD2P4 (see Table 1). This preconditioner is chosen by simply specifying 'ML' as second argument of mld\_precinit (a call to mld\_precset is not needed) and is applied with the BiCGSTAB solver provided by PSBLAS. The setup and application of the default multi-level preconditioners for the real single precision and the complex, single and double precision, versions are obtained with straightforward modifications of the example.

The part of the code concerning the reading and assembling of the sparse matrix and the right-hand side vector, performed through the PSBLAS routines for sparse matrix and vector management, is not reported here for brevity; the statements concerning the deallocation of the PSBLAS data structure are neglected too. The complete code can be found in the example program file example\_ml.f90 in the directory XXXXXX (COMPLETARE. DIRE CHE I FILE IN REALTA' SONO DUE, UNO CON LA GENERAZIONE DELLA MATRICE ED UNO CON LA LETTURA). Note that the modules psb\_base\_mod and psb\_util\_mod at the beginning of the code are required by PSBLAS. O psb\_base\_mod E' RICHIESTO ANCHE DA MLD2P4?) For details on the use of the PSBLAS routines, see the PSBLAS User's Guide [].

#### LE FIGURE SONO DECENTRATE, NONOSTANTE IL CEN-TER. CI VUOLE UNA MINIPAGE?

Different versions of multilevel preconditioner can be obtained by changing the default values of the preconditioner parameters. The code reported in Figure 2 shows how to set a three-level hybrid Schwarz preconditioner, which uses block Jacobi with ILU(0) on the local blocks as post-smoother, a coarsest matrix replicated on the processors, and the LU factorization from UMFPACK as coarse-level solver. The number of levels is specified by using mld\_precinit; the other preconditioner parameters are set by calling mld\_precset. Note that the type of multilevel framework (i.e. multiplicative among the levels with postsmoothing only) is not specified since it is the default set by mld\_precinit. Figure 3 shows how to set a three-level additive Schwarz preconditioner, which applies RAS, with overlap 1 and ILU(0) on the blocks, as pre- and post-smoother, and five block-Jacobi sweeps, with the UMFPACK LU factorization on the blocks, as distributed coarsest-level solver. Again, mld\_precset is used only to set non-default values of the parameters (see Tables ??-??). In both cases, the construction and the application of the preconditioner are carried out as for the default multi-level preconditioner. The code fragments shown in in Figures 2-?? are included in the example program file example\_ml.f90. LO STESSO PROGRAMMA CONTIENE I TRE ESEMPI, CON UN SWITCH TRA L'UNO E L'ALTRO O FACCIAMO 3 PROGRAMMI DISTINTI? NON RICORDO CHE COSA ABBIAMO DECISO.

Finally, Figure 4 shows the setup of a one-level additive Schwarz preconditioner, i.e. RAS with overlap 2. The corresponding code, including also the application of the preconditioner is in the example program file example\_llev.f90.

**Remark.** Any PSBLAS-based program using the basic preconditioners implemented in PSBLAS 2.0, i.e. the diagonal and block-Jacobi ones, can use the diagonal and block-Jacobi preconditioners implemented in MLD2P4 without any change in the code. The PSBLAS-based program must be only recompiled and linked to the MLD2P4 library.

```
use psb_base_mod
 use psb_util_mod
 use mld_prec_mod
 use psb_krylov_mod
. . . . . . .
!
! sparse matrix
 type(psb_dspmat_type) :: A
! sparse matrix descriptor
 type(psb_desc_type) :: desc_A
! preconditioner
 type(mld_dprec_type) :: P
. . . . . . .
!
! initialize the parallel environment
 call psb_init(ictxt)
 call psb_info(ictxt,iam,np)
. . . . . . .
!
! read and assemble the matrix A and the right-hand
! side b using PSBLAS routines for sparse matrix /
! vector management
. . . . . . .
!
! initialize the default multi-level preconditioner,
! i.e. two-level hybrid Schwarz, using RAS (with
! overlap 1 and ILU(0) on the blocks) as post-smoother
! and 4 block-Jacobi sweeps (with UMFPACK LU on the
! blocks) as distributed coarse-level solver
 call mld_precinit(P,'ML',info)
1
! build the preconditioner
 call psb_precbld(A,P,desc_A,info)
1
! set the solver parameters and the initial guess
 ... ...
!
! solve Ax=b with preconditioned BiCGSTAB
 call psb_krylov('BICGSTAB',A,P,b,x,tol,desc_A,info)
 ... ...
1
! deallocate the preconditioner
 call mld_precfree(P,info)
I.
! deallocate other data structures
 ... ...
ţ
! exit the parallel environment
 call psb_exit(ictxt)
 stop
```

Figure 1: Setup and application of the default multi-level Schwarz preconditioner.

```
! set a three-level hybrid Schwarz preconditioner,
! which uses block Jacobi (with ILU(0) on the blocks)
! as post-smoother, a coarsest matrix replicated on the
! processors, and the LU factorization from UMFPACK
! as coarse-level solver
call mld_precinit(P,'ML',info,nlev=3)
call_mld_precset(P,mld_smoother_type_,'BJAC',info)
call mld_precset(P,mld_coarse_mat,'REPL')
call mld_precset(P,mld_coarse_solve,'UMF')
...
```

Figure 2: Setup of a hybrid three-level Schwarz preconditioner.

```
! set a three-level additive Schwarz preconditioner,
! which uses RAS (with overlap 1 and ILU(0) on the blocks)
! as pre- and post-smoother, and 5 block-Jacobi sweeps
! (with UMFPACK LU on the blocks) as distributed
! coarsest-level solver
    call mld_precinit(P,'ML',info,nlev=3)
    call mld_precset(P,mld_ml_type_,'ADD',info)
    call_mld_precset(P,mld_smoother_pos_,'TWOSIDE',info)
    call mld_precset(P,mld_coarse_sweeps_,5)
......
```

Figure 3: Setup of an additive three-level Schwarz preconditioner.

! set RAS with overlap 2 and ILU(0) on the local blocks
 call mld\_precinit(P,'AS',info)
 call mld\_precset(P,mld\_sub\_ovr\_,2,info)
...

Figure 4: Setup of a one-level Schwarz preconditioner.

# 6 User Interface

The basic user interface of MLD2P4 consists of six routines. The four routines mld\_precinit, mld\_precset, mld\_precbld and mld\_precaply encapsulate all the functionalities for the setup and application of any one-level and multi-level preconditioner implemented in the package. The routine mld\_precfree deallocates the preconditioner data structure, while mld\_precdescr prints a description of the preconditioner setup by the user.

For each routine, the same user interface is overloaded with respect to the real/complex case and the single/double precision; arguments with appropriate data types must be passed to the routine, i.e.

- the sparse matrix data structure, containing the matrix to be preconditioned, must be of type mld\_xspmat\_type with x = s for real single precision, x = d for real double precision, x = c for complex single precision, x = z for complex double precision;
- the preconditioner data structure must be of type mld\_xprec\_type, with x = s, d, c, z, according to the sparse matrix data structure;
- the arrays containing the vectors v and w involved in the preconditioner application  $w = M^{-1}v$  must be of type  $type(kind_parameter)$ , with type= real, complex and  $kind_parameter = kind(1.)$ , kind(1.d0), according to the sparse matrix and preconditioner data structure; note that the PSBLAS module provides the constants  $psb_spk_ = kind(1.)$  and  $psb_dpk_ = kind(1.d0)$ ;
- real parameters defining the preconditioner must be declared according to the precision of the previous data structures (see Section 6.2).

A description of each routine is given in the remainder of this section.

#### 6.1 Subroutine mld\_precinit

mld\_precinit(p,ptype,info)
mld\_precinit(p,ptype,info,nlev)

This routine allocates and initializes the preconditioner data structure, according to the preconditioner type chosen by the user.

р	type(mld_xprec_type), intent(inout). CONTROLLARE SE DEVE ESSERE INOUT O SOLO OUT
	The preconditioner data structure. Note that $x$ must be chosen according
	to the real/complex, single/double precision version of MLD2P4 under use.
ptype	character(len=*), intent(in).
	The type of preconditioner. Its values are specified in Table 1.
info	integer, intent(out).
	Error code. See Section 7 for details.
nlev	<pre>integer, optional, intent(in).</pre>
	The number of levels of the multilevel preconditioner. If <b>nlev</b> is not present and <b>ptype</b> ='ML'/'ml', then <b>nlev</b> =2 is assumed. Otherwise, <b>nlev</b> is ignored.

#### 6.2 Subroutine mld\_precset

mld\_precset(p,what,val,info)

This routine sets the parameters defining the preconditioner. More precisely, the parameter identified by what is assigned the value contained in val.

#### Arguments

р	<pre>type(mld_xprec_type), intent(inout).</pre>
	The preconditioner data structure. Note that $x$ must be chosen according
	to the real/complex, single/double precision version of MLD2P4 under use.
what	integer, intent(in).
	The number identifying the parameter to be set. A mnemonic constant has
	been associated to each of these numbers, as reported in Tables 2-5.
val	<pre>integer or character(len=*) or real(kind(1.)) or real(kind(1.d0)),</pre>
	intent(in).
	The value of the parameter to be set. The list of allowed values and the
	corresponding data types is given in Table ??.
info	integer, intent(out).
	Error code. See Section 7 for details.

A variety of (one-level and multi-level) preconditioner can be obtained by a suitable setting of the preconditioner parameters. These parameters can be logically divided into four groups, i.e. parameters defining

- 1. the type of multi-level preconditioner;
- 2. the one-level preconditioner to be used as smoother;
- 3. the aggregation algorithm;
- 4. the coarse-space correction at the coarsest level.

A list of the parameters that can be set, along with their allowed and default values, is given in Tables 2-5. CORREGGERE LA ROUTINE E LA DOC INTERNA - ilev NON E' PIU' ACCESSIBILE ALL'UTENTE.

what	data type	val	default	comments
mld_ml_type_	character(len=*)	'ADD'	'MULT'	basic multi-level framework: additive or mul-
		'MULT'		tiplicative among the levels always additive in-
				side a level)
mld_smoother_type_	character(len=*)	'DIAG'	$\mathbf{AS}^{\prime}$	basic one-level preconditioner (i.e. smoother)
		'BJAC'		of the multi-level preconditioner CAM-
		AS'		BIARE NOME COSTANTE NEL SW,
				ORA E' mld_prec_type. INIBIRE
				no-prec NELL'AMBITO DEL MULTI-
				LEVEL.
mld_smoother_pos_	character(len=*)	'PRE'	$2, \dots, nlev$	"position" of the smoother: pre-smoother,
		POST'		post-smoother, pre-/post-smoother
		'TWOSIDE'		PREFERISCO TWOSIDE A BOTH
				PERCHE' E' DIVERSO DA TRILI-
				NOS

Table 2: Parameters defining the type of multi-level preconditioner.

		CAMBIARE NOME PARAMETRO			CAMBIARE NOME PARAMETRO	MANCA COSTANTE STRINGA AS-	
		NOME			NOME	STANTE	
	$default \mid comments$	CAMBIARE	NEL SW		CAMBIARE	MANCA COS	SOCIATA
-	default	<del>,</del>					
	val	any number	$0 \ge 0$				
	$data \ type$	integer					

mld\_sub\_restr\_ mld\_sub\_prol\_ mld\_sub\_solve\_ mld\_sub\_fillin\_

mld\_sub\_ovr

what

mld\_sub\_thresh\_ mld\_sub\_ren\_ Table 3: Parameters defining the basic one-level preconditioner (smoother).

what	data type val	val	default	default comments	
mld_aggr_alg_					
mld_aggr_kind_					
mld_aggr_thresh_					
mld_aggr_eig_				MANCA STRINC	<b>AANCA STRINGA CORRISPON-</b>
				DENTE a mld_max_norm	norm

Table 4: Parameters defining the aggregation algorithm.

what	data type val	val	default	default   comments
mld_coarse_mat_				
mld_coarse_solve_				VEDI OSSERVAZIONI EMAIL 15-
				16/06/08
mld_coarse_subsolve_				VEDI OSSERVAZIONI EMAIL 15-
				16/06/08
mld_coarse_sweeps_				
mld_coarse_fillin_				MODIFICA NOME PARAM. NEL SW
mld_coarse_thresh_				

Table 5: Parameters defining the coarse-space correction at the coarsest level.

# 6.3 Subroutine mld\_precbld

# mld\_precbld(a,desc\_a,p,info)

This routine builds the preconditioner according to the requirements made by the user through the routines mld\_precinit and mld\_precset.

a	<pre>type(psb_xspmat_type), intent(in).</pre>
	The sparse matrix structure containing the local part of the matrix to be
	preconditioned. Note that $x$ must be chosen according to the real/complex,
	single/double precision version of MLD2P4 under use. See the PSBLAS
	User's Guide for details [?].
desc_a	type(psb_desc_type), intent(in).
	The communication descriptor of a. See the PSBLAS User's Guide for
	details [?].
р	<pre>type(mld_xprec_type), intent(inout).</pre>
	The preconditioner data structure. Note that $x$ must be chosen according
	to the real/complex, single/double precision version of MLD2P4 under use.
info	integer, intent(out).
	Error code. See Section 7 for details.

### 6.4 Subroutine mld\_precaply

mld\_precaply(p,x,y,desc\_a,info)
mld\_precaply(p,x,y,desc\_a,info,trans,work)

This routine computes  $y = op(M^{-1})x$ , where M is a previously built preconditioner, stored in the p data structure, and op denotes the preconditioner itself or its transpose, according to the value of trans. Note that, when MLD2P4 is used with a Krylov solver from PSBLAS, mld\_precaply is called within the PSBLAS routine mld\_krylov and hence is completely transparent to the user.

#### Arguments

р	type(mld_xprec_type), intent(inout). The preconditioner data structure, containing the local part of $M$ . Note that $x$ must be chosen according to the real/complex, single/double precision version of MLD2P4 under use.
x	type(kind_parameter), dimension(:), intent(in). The local part of the vector x. Note that type and kind_parameter must be chosen according to the real/complex, single/double precision version of MLD2P4 under use.
у	type(kind_parameter), dimension(:), intent(out). The local part of the vector y. Note that type and kind_parameter must be chosen according to the real/complex, single/double precision version of MLD2P4 under use.
desc_a	<pre>type(psb_desc_type), intent(in). The communication descriptor associated to the matrix to be precondi- tioned.</pre>
info	integer, intent(out). Error code. See Section 7 for details.
trans	character(len=1), optional, intent(in). If trans = 'N', 'n' then $op(M^{-1}) = M^{-1}$ ; if trans = 'T', 't' then $op(M^{-1}) = M^{-T}$ (transpose of $M^{-1}$ ).
work	<pre>type(kind_parameter), dimension(:), optional, target. Workspace. Its size must be at least 4 * psb_cd_get_local_cols(desc_a) (see the PSBLAS User's Guide). Note that type and kind_parameter must be chosen according to the real/complex, single/double precision version of MLD2P4 under use.</pre>

### 6.5 Subroutine mld\_precfree

#### mld\_precfree(p,info)

This routine deallocates the preconditioner data structure.

р	<pre>type(mld_xprec_type), intent(inout).</pre>
	The preconditioner data structure. Note that $x$ must be chosen according
	to the real/complex, single/double precision version of MLD2P4 under use.
info	integer, intent(out).
	Error code. See Section 7 for details.

# 6.6 Subroutine mld\_precdescr

# mld\_precdescr(p,iout)

This routine prints a description of the preconditioner to the standard output or to a file. FARE UNA SOLA ROUTINE, COL PARAMETRO IOUT OPZIONALE.

<pre>type(mld_xprec_type), intent(in).</pre>
The preconditioner data structure. Note that $x$ must be chosen
according to the real/complex, single/double precision version of
MLD2P4 under use.
integer, intent(in).
The id of the file where the preconditioner description will be printed.
If iout is missing, the description is printed on the standard output.

# 7 Error Handling

Error handling - Breve descrizione con rinvio alla guida di PSBLAS

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