

# PSBLAS-2.0 User's guide

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*A reference guide for the Parallel Sparse BLAS library*

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# 1 Introduction

The PSBLAS library, developed with the aim to facilitate the parallelization of computationally intensive scientific applications, is designed to address parallel implementation of iterative solvers for sparse linear systems through the distributed memory paradigm. It includes routines for multiplying sparse matrices by dense matrices, solving block diagonal systems with triangular diagonal entries, preprocessing sparse matrices, and contains additional routines for dense matrix operations. The current implementation of PSBLAS addresses a distributed memory execution model operating with message passing.

The PSBLAS library is internally implemented in a mixture of Fortran 77 and Fortran 95 [20] programming languages. A similar approach has been advocated by a number of authors, e.g. [19]. Moreover, the Fortran 95 facilities for dynamic memory management and interface overloading greatly enhance the usability of the PSBLAS subroutines. In this way, the library can take care of runtime memory requirements that are quite difficult or even impossible to predict at implementation or compilation time. The following presentation of the PSBLAS library follows the general structure of the proposal for serial Sparse BLAS [15, 16], which in its turn is based on the proposal for BLAS on dense matrices [1, 2, 3].

The applicability of sparse iterative solvers to many different areas causes some terminology problems because the same concept may be denoted through different names depending on the application area. The PSBLAS features presented in this section will be discussed mainly in terms of finite difference discretizations of Partial Differential Equations (PDEs). However, the scope of the library is wider than that: for example, it can be applied to finite element discretizations of PDEs, and even to different classes of problems such as nonlinear optimization, for example in optimal control problems.

The design of a solver for sparse linear systems is driven by many conflicting objectives, such as limiting occupation of storage resources, exploiting regularities in the input data, exploiting hardware characteristics of the parallel platform. To achieve an optimal communication to computation ratio on distributed memory machines it is essential to keep the *data locality* as high as possible; this can be done through an appropriate data allocation strategy. The choice of the preconditioner is another very important factor that affects efficiency of the implemented application. Optimal data distribution requirements for a given preconditioner may conflict with distribution requirements of the rest of the solver. Finding the optimal trade-off may be very difficult because it is application dependent. Possible solution to these problems and other important inputs to the development of the PSBLAS software package has come from an established experience in applying the PSBLAS solvers to computational fluid dynamics applications.

## 2 General overview

The PSBLAS library is designed to handle the implementation of iterative solvers for sparse linear systems on distributed memory parallel computers. The system coefficient matrix  $A$  must be square; it may be real or complex, nonsymmetric, and its sparsity pattern needs not to be symmetric. The serial

computation parts are based on the serial sparse BLAS, so that any extension made to the data structures of the serial kernels is available to the parallel version. The overall design and parallelization strategy have been influenced by the structure of the ScaLAPACK parallel library. The layered structure of the PSBLAS library is shown in figure 1 ; lower layers of the library indicate an encapsulation relationship with upper layers. The ongoing discussion focuses on the Fortran 95 layer immediately below the application layer. The serial parts of the computation on each process are executed through calls to the serial sparse BLAS subroutines. In a similar way, the inter-process message exchanges are implemented through the Basic Linear Algebra Communication Subroutines (BLACS) library [14] that guarantees a portable and efficient communication layer. The Message Passing Interface code is encapsulated within the BLACS layer. However, in some cases, MPI routines are directly used either to improve efficiency or to implement communication patterns for which the BLACS package doesn't provide any method.

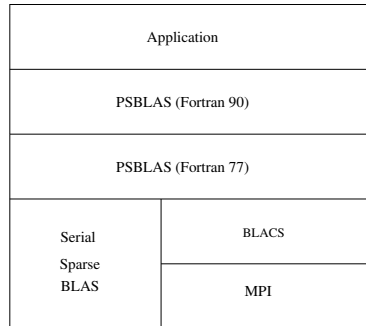


Figure 1: PSBLAS library components hierarchy.

The PSBLAS library consists of various classes of subroutines:

**Computational routines** comprising:

- Sparse matrix by dense matrix product;
- Sparse triangular systems solution for block diagonal matrices;
- Vector and matrix norms;
- Dense matrix sums;
- Dot products.

**Communication routines** handling halo and overlap communications;

**Data management and auxiliary routines** including:

- Parallel environment management
- Communication descriptors allocation;
- Dense and sparse matrix allocation;
- Dense and sparse matrix build and update;
- Sparse matrix and data distribution preprocessing.

## Preconditioner routines

**Iterative methods** a subset of Krylov subspace iterative methods

The following naming scheme has been adopted for all the symbols internally defined in the PSBLAS software package:

- all the symbols (i.e. subroutine names, data types...) are prefixed by `psb_`
- all the data type names are suffixed by `_type`
- all the constant values are suffixed by `_`
- all the subroutine names follow the rule `psb_xxname` where `xx` can be either:
  - `ge`: the routine is related to dense data,
  - `sp`: the routine is related to sparse data,
  - `cd`: the routine is related to communication descriptor (see 3).

For example the `psb_geins`, `psb_spins` and `psb_cdins` perform the same action (see 6) on dense matrices, sparse matrices and communication descriptors respectively. Interface overloading allows the usage of the same subroutine interfaces for both real and complex data.

In the description of the subroutines, arguments or argument entries are classified as:

**global** For input arguments, the value must be the same on all processes participating in the subroutine call; for output arguments the value is guaranteed to be the same.

**local** Each process has its own value(s) independently.

## 2.1 Application structure

The main underlying principle of the PSBLAS library is that the library objects are created and exist with reference to a discretized space to which there corresponds an index space and a matrix sparsity pattern. As an example, consider a cell-centered finite-volume discretization of the Navier-Stokes equations on a simulation domain; the index space  $1 \dots n$  is isomorphic to the set of cell centers, whereas the pattern of the associated linear system matrix is isomorphic to the adjacency graph imposed on the discretization mesh by the discretization stencil.

Thus the first order of business is to establish an index space, and this is done with a call to `psb_cdall` in which we specify the size of the index space  $n$  and the allocation of the elements of the index space to the various processes making up the MPI (virtual) parallel machine.

The index space is partitioned among processes, and this creates a mapping from the “global” numbering  $1 \dots n$  to a numbering “local” to each process; each process  $i$  will own a certain subset  $1 \dots n_{\text{row}_i}$ , each element of which corresponds to a certain element of  $1 \dots n$ . The user does not set explicitly this mapping; when the application needs to indicate to which element of the index space a

certain item is related, such as the row and column index of a matrix coefficient, it does so in the “global” numbering, and the library will translate into the appropriate “local” numbering.

For a given index space  $1 \dots n$  there are many possible associated topologies, i.e. many different discretization stencils; thus the description of the index space is not completed until the user has defined a sparsity pattern, either explicitly through `psb_cdins` or implicitly through `psb_spins`. The descriptor is finalized with a call to `psb_cdasb` and a sparse matrix with a call to `psb_spasb`. After `psb_cdasb` each process  $i$  will have defined a set of “halo” (or “ghost”) indices  $n_{\text{row}_i} + 1 \dots n_{\text{col}_i}$ , denoting elements of the index space that are *not* assigned to process  $i$ ; however the variables associated with them are needed to complete computations associated with the sparse matrix  $A$ , and thus they have to be fetched from (neighbouring) processes. The descriptor of the index space is built exactly for the purpose of properly sequencing the communication steps required to achieve this objective.

A simple application structure will walk through the index space allocation, matrix/vector creation and linear system solution as follows:

1. Initialize parallel environment with `psb_init`
2. Initialize index space with `psb_cdall`
3. Allocate sparse matrix and dense vectors with `psb_spall` and `psb_geall`
4. Loop over all local rows, generate matrix and vector entries, and insert them with `psb_spins` and `psb_geins`
5. Assemble the various entities:
  - (a) `psb_cdasb`
  - (b) `psb_spasb`
  - (c) `psb_geasb`
6. Choose the preconditioner to be used with `psb_precset` and build it with `psb_precbld`
7. Call the iterative method of choice, e.g. `psb_bicgstab`

This is the structure of the sample program `test/pargen/ppde90.f90`.

For a simulation in which the same discretization mesh is used over multiple time steps, the following structure may be more appropriate:

1. Initialize parallel environment with `psb_init`
2. Initialize index space with `psb_cdall`
3. Loop over the topology of the discretization mesh and build the descriptor with `psb_cdins`
4. Assemble the descriptor with `psb_cdasb`
5. Allocate the sparse matrices and dense vectors with `psb_spall` and `psb_geall`
6. Loop over the time steps:

- (a) If after first time step, reinitialize the sparse matrix with `psb_sprn`; also zero out the dense vectors;
- (b) Loop over the mesh, generate the coefficients and insert/update them with `psb_spins` and `psb_geins`
- (c) Assemble with `psb_spasb` and `psb_geasb`
- (d) Choose and build preconditioner with `psb_precset` and `psb_precbld`
- (e) Call the iterative method of choice, e.g. `psb_bicgstab`

The insertion routines will be called as many times as needed; it is clear that they only need be called on the data that is actually allocated to the current process, i.e. each process generates its own data.

In principle there is no specific order in the calls to `psb_spins`, nor is there a requirement to build a matrix row in its entirety before calling the routine; this allows the application programmer to walk through the discretization mesh element by element, generating the main part of a given matrix row but also contributions to the rows corresponding to neighbouring elements.

From a functional point of view it is even possible to execute one call for each nonzero coefficient; however this would have a substantial computational overhead. It is therefore advisable to pack a certain amount of data into each call to the insertion routine, say touching on a few tens of rows; the best performing value would depend on both the architecture of the computer being used and on the problem structure. At the opposite extreme, it would be possible to generate the entire part of a coefficient matrix residing on a process and pass it in a single call to `psb_spins`; this, however, would entail a doubling of memory occupation, and thus would be almost always far from optimal.

## 2.2 Programming model

The PSBLAS library is based on the Single Program Multiple Data (SPMD) programming model: each process participating in the computation performs the same actions on a chunk of data. Parallelism is thus data-driven.

Because of this structure, practically all subroutines *must* be called simultaneously by all processes participating in the computation, i.e each subroutine call acts implicitly as a synchronization point. The exceptions to this rule are:

- The insertion routines `psb_cdins`, `psb_spins` and `psb_geins`;
- The error handling routines.

In particular, as per the discussion in the previous section, the insertion routines may be called a different number of times on each process, depending on the data distribution chosen by the user.



## 3 Data Structures

In this chapter are illustrated data structures used for definition of routines interfaces. This include data structure for sparse matrix, communication descriptor and preconditioner. These data structures are used for calling PSBLAS routines in Fortran 90 language and will be used to next chapters containing these callings.

All the data types and subroutine interfaces are defined in the module `psb_sparse_mod`; this will have to be included by every user subroutine that makes use of the library.

### 3.1 Descriptor data structure

All the general matrix informations and elements to be exchanged among processes are stored within a data structure of the type `psb_desc_type`. Every structure of this type is associated to a sparse matrix, it contains data about general matrix informations and elements to be exchanged among processes.

It is not necessary for the user to know the internal structure of `psb_desc_type`, it is set in fully-transparent mode by PSBLAS-TOOLS routines when inserting a new sparse matrix, however the definition of the descriptor is the following.

**matrix\_data** includes general information about matrix and process grid. More precisely:

**matrix\_data[psb\_dec\_type\_]** Identifies the decomposition type (global); the actual values are internally defined, so they should never be accessed directly.

**matrix\_data[psb\_ctxt\_]** Communication context associated with the processes comprised in the virtual parallel machine (global).

**matrix\_data[psb\_m\_]** Total number of equations (global).

**matrix\_data[psb\_n\_]** Total number of variables (global).

**matrix\_data[psb\_n\_row\_]** Number of grid variables owned by the current process (local); equivalent to the number of local rows in the sparse coefficient matrix.

**matrix\_data[psb\_n\_col\_]** Total number of grid variables read by the current process (local); equivalent to the number of local columns in the sparse coefficient matrix. They include the halo.

Specified as: a pointer to integer array of dimension 10.

**halo\_index** A list of the halo and boundary elements for the current process to be exchanged with other processes; for each processes with which it is necessary to communicate:

1. Process identifier;
2. Number of points to be received;
3. Indices of points to be received;
4. Number of points to be sent;
5. Indices of points to be sent;

The list may contain an arbitrary number of groups; its end is marked by a -1.

Specified as: a pointer to an integer array of rank one.

**overlap\_index** A list of the overlap elements for the current process, organized in groups like the previous vector:

1. Process identifier;
2. Number of points to be received;
3. Indices of points to be received;
4. Number of points to be sent;
5. Indices of points to be sent;

The list may contain an arbitrary number of groups; its end is marked by a -1.

Specified as: a pointer to an integer array of rank one.

**overlap\_index** For all overlap points belonging to the current process:

1. Overlap point index;
2. Number of processes sharing that overlap points;

The list may contain an arbitrary number of groups; its end is marked by a -1.

Specified as: a pointer to an integer array of rank one.

**loc\_to\_glob** each element  $i$  of this array contains global identifier of the local variable  $i$ .

Specified as: a pointer to an integer array of rank one.

**glob\_to\_loc** if global variable  $i$  is read by current process then element  $i$  contains local index correspondent to global variable  $i$ ; else element  $i$  contains -1 (NULL) value.

Specified as: a pointer to an integer array of rank one.

FORTTRAN95 interface for **psb\_desc\_type** structures is therefore defined as follows:

```
type psb_desc_type
  integer, pointer :: matrix_data(:), halo_index(:)
  integer, pointer :: overlap_elem(:), overlap_index(:)
  integer, pointer :: loc_to_glob(:), glob_to_loc(:)
end type psb_desc_type
```

Figure 2: The PSBLAS defined data type that contains the communication descriptor.

A communication descriptor associated with a sparse matrix has a state, which can take the following values:

**Build:** State entered after the first allocation, and before the first assembly; in this state it is possible to add communication requirements among different processes.

**Assembled:** State entered after the assembly; computations using the associated sparse matrix, such as matrix-vector products, are only possible in this state.

### 3.1.1 Named Constants

**psb\_none\_** Generic no-op;

**psb\_nohalo\_** Do not fetch halo elements;

**psb\_halo\_** Fetch halo elements from neighbouring processes;

**psb\_sum\_** Sum overlapped elements

**psb\_avg\_** Average overlapped elements

**psb\_dec\_type\_** Entry holding decomposition type (in `desc_a%matrix_data`)

**psb\_m\_** Entry holding total number of rows

**psb\_n\_** Entry holding total number of columns

**psb\_n\_row\_** Entry holding the number of rows stored in the current process

**psb\_n\_col\_** Entry holding the number of columns stored in the current process

**psb\_ctxt\_** Entry holding a copy of the BLACS communication context

**psb\_desc\_asb\_** State of the descriptor: assembled, i.e. suitable for computational tasks.

**psb\_desc\_bld\_** State of the descriptor: build, must be assembled before computational use.

## 3.2 Sparse Matrix data structure

The `psb_spmat_type` data structure contains all information about local portion of the sparse matrix and its storage mode. Many of these fields are set in fully-transparent mode by PSBLAS-TOOLS routines when inserting a new sparse matrix, user must set only fields which describe matrix storage mode.

Fields contained in Sparse matrix structures are:

**aspk** Contains values of the local distributed sparse matrix.

Specified as: a pointer to an array of rank one of type corresponding to matrix entries type.

**ia1** Holds integer information on distributed sparse matrix. Actual information will depend on data format used.

Specified as: a pointer to an integer array of rank one.

- ia2** Holds integer information on distributed sparse matrix. Actual information will depend on data format used.  
Specified as: a pointer to an integer array of rank one.
- infoa** On entry can hold auxiliary information on distributed sparse matrix. Actual information will depend on data format used.  
Specified as: integer array of length `psb_ifasize_`.
- fida** Defines the format of the distributed sparse matrix.  
Specified as: a string of length 5
- descra** Describe the characteristic of the distributed sparse matrix.  
Specified as: array of character of length 9.
- pl** Specifies the local row permutation of distributed sparse matrix. If `pl(1)` is equal to 0, then there isn't row permutation.  
Specified as: pointer to integer array of dimension equal to number of local row (`matrix_data[psb_n_row_]`)
- pr** Specifies the local column permutation of distributed sparse matrix. If `PR(1)` is equal to 0, then there isn't column permutation.  
Specified as: pointer to integer array of dimension equal to number of local row (`matrix_data[psb_n_col_]`)
- m** Number of rows; if row indices are stored explicitly, as in Coordinate Storage, should be greater than or equal to the maximum row index actually present in the sparse matrix. Specified as: integer variable.
- k** Number of columns; if column indices are stored explicitly, as in Coordinate Storage or Compressed Sparse Rows, should be greater than or equal to the maximum column index actually present in the sparse matrix. Specified as: integer variable.

FORTTRAN95 interface for distributed sparse matrices containing double precision real entries is defined as in figure 3.

```

type psb_dspmat_type
  integer      :: m, k
  character    :: fida(5)
  character    :: descra(10)
  integer      :: infoa(psb_ifa_size_)
  real(kind(1.d0)), pointer :: aspk(:)
  integer, pointer :: ia1(:), ia2(:), pr(:), pl(:)
end type psb_dspmat_type

```

Figure 3: The PSBLAS defined data type that contains a sparse matrix.

The following two cases are among the most commonly used:

**fida**=“**CSR**” Compressed storage by rows. In this case the following should hold:

1. `ia2(i)` contains the index of the first element of row `i`; the last element of the sparse matrix is thus stored at index  $ia2(m+1) - 1$ . It should contain `m+1` entries in nondecreasing order (strictly increasing, if there are no empty rows).
2. `ia1(j)` contains the column index and `aspk(j)` contains the corresponding coefficient value, for all  $ia2(1) \leq j \leq ia2(m+1) - 1$ .

**fida=“COO”** Coordinate storage. In this case the following should hold:

1. `infoa(1)` contains the number of nonzero elements in the matrix;
2. For all  $1 \leq j \leq infoa(1)$ , the coefficient, row index and column index are stored into `aspk(j)`, `ia1(j)` and `ia2(j)` respectively.

A sparse matrix has an associated state, which can take the following values:

**Build:** State entered after the first allocation, and before the first assembly; in this state it is possible to add nonzero entries.

**Assembled:** State entered after the assembly; computations using the sparse matrix, such as matrix-vector products, are only possible in this state;

**Update:** State entered after a reinitialization; this is used to handle applications in which the same sparsity pattern is used multiple times with different coefficients. In this state it is only possible to enter coefficients for already existing nonzero entries.

### 3.2.1 Named Constants

`psb_dupl_ovwrt_` Duplicate coefficients should be overwritten (i.e. ignore duplications)

`psb_dupl_add_` Duplicate coefficients should be added;

`psb_dupl_err_` Duplicate coefficients should trigger an error conditino

`psb_upd_dflt_` Default update strategy for matrix coefficients;

`psb_upd_srch_` Update strategy based on search into the data structure;

`psb_upd_perm_` Update strategy based on additional permutation data (see tools routine description).

## 3.3 Preconditioner data structure

PSBLAS-2.0 offers the possibility to use many different types of preconditioning schemes. Besides the simple well known preconditioners like Diagonal Scaling or Block Jacobi (with ILU(0) incomplete factorization) also more complex preconditioning methods are implemented like the Additive Schwarz and Two-Level ones. A preconditioner is held in the `psb_prec_type` data structure which depends on the `psb_base_prec` reported in figure 4. The `psb_base_prec` data type may contain a simple preconditioning matrix with the associated communication descriptor which may be different than the system communication descriptor in the case of parallel preconditioners like the Additive Schwarz one.

Then the `psb_prec_type` may contain more than one preconditioning matrix like in the case of Two-Level (in general Multi-Level) preconditioners. The user can choose the type of preconditioner to be used by means of the `psb_precset` subroutine; once the type of preconditioning method is specified, along with all the parameters that characterize it, the preconditioner data structure can be built using the `psb_precbld` subroutine. This data structure wants to be flexible enough to easily allow the implementation of new kind of preconditioners. The values contained in the `iprcparm` and `dprcparm` define the type of preconditioner along with all the parameters related to it; thus, `iprcparm` and `dprcparm` define how the other records have to be interpreted.

```

type psb_base_prec

  type(psb_spmat_type), pointer  :: av(:) => null()
  real(kind(1.d0)), pointer     :: d(:) => null()
  type(psb_desc_type), pointer  :: desc_data => null()
  integer, pointer              :: iprcparm(:) => null()
  real(kind(1.d0)), pointer     :: dprcparm(:) => null()
  integer, pointer              :: perm(:) => null()
  integer, pointer              :: mlia(:) => null()
  integer, pointer              :: invperm(:) => null()
  integer, pointer              :: nlaggr(:) => null()
  type(psb_spmat_type), pointer :: aorig  => null()
  real(kind(1.d0)), pointer     :: dorig(:) => null()

end type psb_base_prec

type psb_prec_type
  type(psb_base_prec), pointer  :: baseprecv(:) => null()
  integer                       :: prec, base_prec
end type psb_prec_type

```

Figure 4: The PSBLAS defined data type that contains a preconditioner.

### 3.3.1 Named Constants

**f\_ilu\_n\_** Incomplete LU factorization with  $n$  levels of fill-in; currently only  $n = 0$  is implemented;

**f\_slu\_** Sparse factorization using SuperLU;

**f\_umf\_** Sparse factorization using UMFPACK;

**add\_ml\_prec\_** Additive multilevel correction;

**mult\_ml\_prec\_** Multiplicative multilevel correction;

**pre\_smooth\_** Pre-smoothing in applying multiplicative multilevel corrections;

**post\_smooth\_** Post-smoothing in applying multiplicative multilevel corrections;

**smooth\_both\_** Two-sided (i.e. symmetric) smoothing in applying multiplicative multilevel corrections;

**mat\_distr\_** Coarse matrix distributed among processes

**mat\_repl\_** Coarse matrix replicated among processes

## 4 Computational routines



## psb\_geaxpby—General Dense Matrix Sum

This subroutine is an interface to the computational kernel for dense matrix sum:

$$y \leftarrow \alpha x + \beta y$$

### Syntax

call psb\_geaxpby (*alpha*, *x*, *beta*, *y*, *desc\_a*, *info*)

<i>x</i> , <i>y</i> , $\alpha$ , $\beta$	Subroutine
Long Precision Real	psb_geaxpby
Long Precision Complex	psb_geaxpby

Table 1: Data types

### On Entry

- alpha** the scalar  $\alpha$ .  
Scope: **global**  
Type: **required**  
Specified as: a number of the data type indicated in Table 1.
- x** the local portion of global dense matrix  $x$ .  
Scope: **local**  
Type: **required**  
Specified as: a rank one or two array containing numbers of type specified in Table 1. The rank of  $x$  must be the same of  $y$ .
- beta** the scalar  $\beta$ .  
Scope: **global**  
Type: **required**  
Specified as: a number of the data type indicated in Table 1.
- y** the local portion of the global dense matrix  $y$ .  
Scope: **local**  
Type: **required**  
Specified as: a rank one or two array containing numbers of the type indicated in Table 1. The rank of  $y$  must be the same of  $x$ .
- desc\_a** contains data structures for communications.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type [psb\\_desc\\_type](#).

### On Return

**y** the local portion of result submatrix  $y$ .

Scope: **local**

Type: **required**

Specified as: a rank one or two array containing numbers of the type indicated in [Table 1](#).

**info** the local portion of result submatrix  $y$ .

Scope: **local**

Type: **required**

An integer value that contains an error code.

## psb\_gedot—Dot Product

This function computes dot product between two vectors  $x$  and  $y$ .  
If  $x$  and  $y$  are double precision real vectors computes dot-product as:

$$dot \leftarrow x^T y$$

Else if  $x$  and  $y$  are double precision complex vectors then computes dot-product as:

$$dot \leftarrow x^H y$$

## Syntax

`psb_gedot (x, y, desc_a, info)`

<i>dot, x, y</i>	<b>Function</b>
Long Precision Real	<code>psb_gedot</code>
Long Precision Complex	<code>psb_gedot</code>

Table 2: Data types

### On Entry

**x** the local portion of global dense matrix  $x$ .

Scope: **local**

Type: **required**

Specified as: an array of rank one or two containing numbers of type specified in Table 2. The rank of  $x$  must be the same of  $y$ .

**y** the local portion of global dense matrix  $y$ .

Scope: **local**

Type: **required**

Specified as: an array of rank one or two containing numbers of type specified in Table 2. The rank of  $y$  must be the same of  $x$ .

**desc\_a** contains data structures for communications.

Scope: **local**

Type: **required**

Specified as: a structured data of type `psb_desc_type`.

### On Return

**Function value** is the dot product of subvectors  $x$  and  $y$ .

Scope: **global**

Specified as: a number of the data type indicated in Table 2.

**info** the local portion of result submatrix  $y$ .  
Scope: **local**  
Type: **required**  
An integer value that contains an error code.

## psb\_gedot—Generalized Dot Product

This subroutine computes a series of dot products among the columns of two dense matrices  $x$  and  $y$ :

$$res(i) \leftarrow x(:,i)^T y(:,i)$$

If the matrices are complex, then the usual convention applies, i.e. the conjugate transpose of  $x$  is used. If  $x$  and  $y$  are of rank one, then  $res$  is a scalar, else it is a rank one array.

### Syntax

psb\_gedot ( $res, x, y, desc\_a, info$ )

$res, x, y$	Subroutine
Long Precision Real	psb_gedot
Long Precision Complex	psb_gedot

Table 3: Data types

#### On Entry

**x** the local portion of global dense matrix  $x$ .

Scope: **local**

Type: **required**

Specified as: an array of rank one or two containing numbers of type specified in Table 3. The rank of  $x$  must be the same of  $y$ .

**y** the local portion of global dense matrix  $y$ .

Scope: **local**

Type: **required**

Specified as: an array of rank one or two containing numbers of type specified in Table 3. The rank of  $y$  must be the same of  $x$ .

**desc\_a** contains data structures for communications.

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_desc\\_type](#).

#### On Return

**res** is the dot product of subvectors  $x$  and  $y$ .

Scope: **global**

Specified as: a number or a rank-one array of the data type indicated in Table 2.

**info** Scope: **local**  
Type: **required**  
An integer value that contains an error code.

## psb\_geamax—Infinity-Norm of Vector

This function computes the infinity-norm of a vector  $x$ .

If  $x$  is a double precision real vector computes infinity norm as:

$$amax \leftarrow \max_i |x_i|$$

else if  $x$  is a double precision complex vector then computes infinity-norm as:

$$amax \leftarrow \max_i (|re(x_i)| + |im(x_i)|)$$

### Syntax

psb\_geamax ( $x$ ,  $desc\_a$ ,  $info$ )

$amax$	$x$	Function
Long Precision Real	Long Precision Real	psb_geamax
Long Precision Real	Long Precision Complex	psb_geamax

Table 4: Data types

#### On Entry

**x** the local portion of global dense matrix  $x$ .

Scope: **local**

Type: **required**

Specified as: a rank one or two array containing numbers of type specified in Table 4.

**desc\_a** contains data structures for communications.

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_desc\\_type](#).

#### On Return

**Function value** is the infinity norm of subvector  $x$ .

Scope: **global**

Specified as: a long precision real number.

**info** Scope: **global**

Type: **required**

An integer value that contains an error code.

## psb\_geamax—Generalized Infinity Norm

This subroutine computes a series of infinity norms on the columns of a dense matrix  $x$ :

$$res(i) \leftarrow \max_k |x(k, i)|$$

### Syntax

psb\_geamax (*res*, *x*, *desc\_a*, *info*)

<i>res</i>	<i>x</i>	Subroutine
Long Precision Real	Long Precision Real	psb_geamax
Long Precision Real	Long Precision Complex	psb_geamax

Table 5: Data types

#### On Entry

**x** the local portion of global dense matrix  $x$ .

Scope: **local**

Type: **required**

Specified as: a rank one or two array containing numbers of type specified in Table 5.

**desc\_a** contains data structures for communications.

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_desc\\_type](#).

#### On Return

**res** is the infinity norm of the columns of  $x$ .

Scope: **global**

Specified as: a number or a rank-one array of long precision real numbers.

**info** Scope: **local**

Type: **required**

An integer value that contains an error code.



## psb\_geasum—1-Norm of Vector

This function computes the 1-norm of a vector  $x$ .

If  $x$  is a double precision real vector computes 1-norm as:

$$asum \leftarrow \|x_i\|$$

else if  $x$  is double precision complex vector then computes 1-norm as:

$$asum \leftarrow \|re(x)\|_1 + \|im(x)\|_1$$

### Syntax

`psb_geasum (x, desc_a, info)`

<i>asum</i>	<i>x</i>	<b>Function</b>
Long Precision Real	Long Precision Real	psb_geasum
Long Precision Real	Long Precision Complex	psb_geasum

Table 6: Data types

#### On Entry

**x** the local portion of global dense matrix  $x$ .

Scope: **local**

Type: **required**

Specified as: a rank one or two array containing numbers of type specified in Table 6.

**desc\_a** contains data structures for communications.

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_desc\\_type](#).

#### On Return

**Function value** is the 1-norm of vector  $x$ .

Scope: **global**

Specified as: a long precision real number.

**info** Scope: **local**

Type: **required**

An integer value that contains an error code.

## psb\_genrm2—2-Norm of Vector

This function computes the 2-norm of a vector  $x$ .

If  $x$  is a double precision real vector computes 2-norm as:

$$nrm2 \leftarrow \sqrt{x^T x}$$

else if  $x$  is double precision complex vector then computes 2-norm as:

$$nrm2 \leftarrow \sqrt{x^H x}$$

$nrm2$	$x$	Function
Long Precision Real	Long Precision Real	psb_genrm2
Long Precision Real	Long Precision Complex	psb_genrm2

Table 7: Data types

## Syntax

`psb_genrm2 (x, desc_a, info)`

### On Entry

**x** the local portion of global dense matrix  $x$ .

Scope: **local**

Type: **required**

Specified as: a rank one or two array containing numbers of type specified in Table 7.

**desc\_a** contains data structures for communications.

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_desc\\_type](#).

### On Return

**Function Value** is the 2-norm of subvector  $x$ .

Scope: **global**

Type: **required**

Specified as: a long precision real number.

**info** Scope: **local**

Type: **required**

An integer value that contains an error code.

## psb\_spnrmi—Infinity Norm of Sparse Matrix

This function computes the infinity-norm of a matrix  $A$ :

$$nrmi \leftarrow \|A\|_{\infty}$$

where:

$A$  represents the global matrix  $A$

$A$	Function
Long Precision Real	psb_spnrmi
Long Precision Complex	psb_spnrmi

Table 8: Data types

### Syntax

`psb_spnrmi (A, desc_a, info)`

#### On Entry

**a** the local portion of the global sparse matrix  $A$ .

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_spmat\\_type](#).

**desc\_a** contains data structures for communications.

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_desc\\_type](#).

#### On Return

**Function value** is the infinity-norm of sparse submatrix  $A$ .

Scope: **global**

Specified as: a long precision real number.

**info** Scope: **local**

Type: **required**

An integer value that contains an error code.

## psb\_spmmm—Sparse Matrix by Dense Matrix Product

This subroutine computes the Sparse Matrix by Dense Matrix Product:

$$y \leftarrow \alpha P_r A P_c x + \beta y \quad (1)$$

$$y \leftarrow \alpha P_r A^T P_c x + \beta y \quad (2)$$

$$y \leftarrow \alpha P_r A^H P_c x + \beta y \quad (3)$$

where:

$x$  is the global dense submatrix  $x_{:,}$ .

$y$  is the global dense submatrix  $y_{:,}$ .

$A$  is the global sparse submatrix  $A$

$P_r, P_c$  are the permutation matrices.

$A, x, y, \alpha, \beta$	Subroutine
Long Precision Real	psb_spmmm
Long Precision Complex	psb_spmmm

Table 9: Data types

## Syntax

CALL psb\_spmmm (*alpha, a, x, beta, y, desc\_a, info*)

CALL psb\_spmmm (*alpha, a, x, beta, y, desc\_a, info, trans, work*)

### On Entry

**alpha** the scalar  $\alpha$ .

Scope: **global**

Type: **required**

Specified as: a number of the data type indicated in Table 9.

**a** the local portion of the sparse matrix  $A$ .

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_spmat\\_type](#).

**x** the local portion of global dense matrix  $x$ .

Scope: **local**

Type: **required**

Specified as: a rank one or two array containing numbers of type specified in Table 9. The rank of  $x$  must be the same of  $y$ .

**beta** the scalar  $\beta$ .  
 Scope: **global**  
 Type: **required**  
 Specified as: a number of the data type indicated in Table 9.

**y** the local portion of global dense matrix  $y$ .  
 Scope: **local**  
 Type: **required**  
 Specified as: a rank one or two array containing numbers of type specified in Table 9. The rank of  $y$  must be the same of  $x$ .

**desc.a** contains data structures for communications.  
 Scope: **local**  
 Type: **required**  
 Specified as: a structured data of type `psb.desc_type`.

**trans** indicate what kind of operation to perform.

**trans** = **N** the operation is specified by equation 1  
**trans** = **T** the operation is specified by equation 2  
**trans** = **C** the operation is specified by equation 3

Scope: **global**  
 Type: **optional**  
 Default:  $trans = N$   
 Specified as: a character variable.

**work** work array.  
 Scope: **local**  
 Type: **optional**  
 Specified as: a rank one array of the same type of  $x$  and  $y$  with the TARGET attribute.

### On Return

**y** the local portion of result submatrix  $y$ .  
 Scope: **local**  
 Type: **required**  
 Specified as: an array of rank one or two containing numbers of type specified in Table 9.

**info** Scope: **local**  
 Type: **required**  
 An integer value that contains an error code.

## psb\_spsm—Triangular System Solve

This subroutine computes the Triangular System Solve:

$$\begin{aligned}
 y &\leftarrow \alpha P_r T^{-1} P_c x + \beta y \\
 y &\leftarrow \alpha D P_r T^{-1} P_c x + \beta y \\
 y &\leftarrow \alpha P_r T^{-1} P_c D x + \beta y \\
 y &\leftarrow \alpha P_r T^{-T} P_c x + \beta y \\
 y &\leftarrow \alpha D P_r T^{-T} P_c x + \beta y \\
 y &\leftarrow \alpha P_r T^{-T} P_c D x + \beta y \\
 y &\leftarrow \alpha P_r T^{-H} P_c x + \beta y \\
 y &\leftarrow \alpha D P_r T^{-H} P_c x + \beta y \\
 y &\leftarrow \alpha P_r T^{-H} P_c D x + \beta y
 \end{aligned}$$

where:

$x$  is the global dense submatrix  $x_{:,}$ .

$y$  is the global dense submatrix  $y_{:,}$ .

$T$  is the global sparse block triangular submatrix  $T$ .

$D$  is the scaling diagonal matrix.

$P_r, P_c$  are the permutation matrices.

### Syntax

CALL psb\_spsm (*alpha, t, x, beta, y, desc\_a, info*)

CALL psb\_spsm

(*alpha, t, x, beta, y, desc\_a, info, trans, unit, choice, diag, work*)

$T, x, y, D, \alpha, \beta$	Subroutine
Long Precision Real	psb_spsm
Long Precision Complex	psb_spsm

Table 10: Data types

### On Entry

**alpha** the scalar  $\alpha$ .  
 Scope: **global**  
 Type: **required**  
 Specified as: a number of the data type indicated in Table 10.

**t** the global portion of the sparse matrix  $T$ .  
 Scope: **local**  
 Type: **required**  
 Specified as: a structured data type specified in § 3.

**x** the local portion of global dense matrix  $x$ .  
 Scope: **local**  
 Type: **required**  
 Specified as: a rank one or two array containing numbers of type specified in Table 10. The rank of  $x$  must be the same of  $y$ .

**beta** the scalar  $\beta$ .  
 Scope: **global**  
 Type: **required**  
 Specified as: a number of the data type indicated in Table 10.

**y** the local portion of global dense matrix  $y$ .  
 Scope: **local**  
 Type: **required**  
 Specified as: a rank one or two array containing numbers of type specified in Table 10. The rank of  $y$  must be the same of  $x$ .

**desc.a** contains data structures for communications.  
 Scope: **local**  
 Type: **required**  
 Specified as: a structured data of type `psb.desc.type`.

**trans** specify with *unitd* the operation to perform.  
**trans** = 'N' the operation is with no transposed matrix  
**trans** = 'T' the operation is with transposed matrix.  
**trans** = 'C' the operation is with conjugate transposed matrix.  
 Scope: **global**  
 Type: **optional**  
 Default: *trans* =  $N$   
 Specified as: a character variable.

**unitd** specify with *trans* the operation to perform.  
**unitd** = 'U' the operation is with no scaling  
**unitd** = 'L' the operation is with left scaling  
**unitd** = 'R' the operation is with right scaling.  
 Scope: **global**  
 Type: **optional**  
 Default: *unitd* =  $U$   
 Specified as: a character variable.

**choice** specifies the update of overlap elements to be performed on exit:

psb\_none\_  
psb\_sum\_  
psb\_avg\_  
psb\_square\_root\_

Scope: **global**  
Type: **optional**  
Default: **psb\_avg\_**  
Specified as: an integer variable.

**diag** the diagonal scaling matrix.

Scope: **local**  
Type: **optional**  
Default:  $diag(1) = 1(noscaling)$   
Specified as: a rank one array containing numbers of the type indicated in Table 10.

**work** a work array.

Scope: **local**  
Type: **optional**  
Specified as: a rank one array of the same type of  $x$  with the TARGET attribute.

### On Return

**y** the local portion of global dense matrix  $y$ .

Scope: **local**  
Type: **required**  
Specified as: a pointer to array of rank one or two containing numbers of type specified in Table 10.

**info** Scope: **local**

Type: **required**  
An integer value that contains an error code.



## 5 Communication routines

The routines in this chapter implement various global communication operators on vectors associated with a discretization mesh. For auxiliary communication routines not tied to a discretization space see [6](#).

## psb\_halo—Halo Data Communication

These subroutines restore a consistent status for the halo elements, and (optionally) scale the result:

$$x \leftarrow \alpha x$$

where:

$x$  is a global dense submatrix.

$\alpha, x$	Subroutine
Long Precision Real	psb_halo
Long Precision Complex	psb_halo

Table 11: Data types

## Syntax

CALL psb\_halo ( $x, desc\_a, info$ )

CALL psb\_halo ( $x, desc\_a, info, alpha, work$ )

### On Entry

**x** global dense matrix  $x$ .

Scope: **local**

Type: **required**

Specified as: a rank one or two array with the TARGET attribute containing numbers of type specified in Table 11.

**desc\_a** contains data structures for communications.

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_desc\\_type](#).

**alpha** the scalar  $\alpha$ .

Scope: **global**

Type: **optional**

Default:  $alpha = 1$

Specified as: a number of the data type indicated in Table 11.

**work** the work array.

Scope: **local**

Type: **optional**

Specified as: a rank one array of the same type of  $x$  with the POINTER attribute.

### On Return

**x** global dense result matrix  $x$ .

Scope: **local**

Type: **required**

Returned as: a rank one or two array containing numbers of type specified in Table 11.

**info** the local portion of result submatrix  $y$ .

Scope: **local**

Type: **required**

An integer value that contains an error code.

## psb\_ovrl—Overlap Update

These subroutines restore a consistent status for the overlap elements:

$$x \leftarrow Qx$$

where:

$x$  is the global dense submatrix  $x$

$Q$  is the overlap operator; it is the composition of two operators  $P_a$  and  $P^T$ .

$x$	Subroutine
Long Precision Real	psb_ovrl
Long Precision Complex	psb_ovrl

Table 12: Data types

## Syntax

CALL psb\_ovrl ( $x$ , desc\_a, info)

CALL psb\_ovrl ( $x$ , desc\_a, info, update=update\_type, work=work)

### On Entry

**x** global dense matrix  $x$ .

Scope: **local**

Type: **required**

Specified as: a rank one or two array containing numbers of type specified in Table 12.

**desc\_a** contains data structures for communications.

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_desc\\_type](#).

**update** Update operator.

**update = psb\_none\_** Do nothing;

**update = psb\_add\_** Sum overlap entries, i.e. apply  $P^T$ ;

**update = psb\_avg\_** Average overlap entries, i.e. apply  $P_a P^T$ ;

Scope: **global**

Default:  $update\_type = psb\_avg\_$

Scope: **global**

Specified as: a integer variable.

**work** the work array.  
Scope: **local**  
Type: **optional**  
Specified as: a one dimensional array of the same type of  $x$ .

### On Return

**x** global dense result matrix  $x$ .  
Scope: **local**  
Type: **required**  
Specified as: an array of rank one or two containing numbers of type specified in Table 12.

**info** the local portion of result submatrix  $y$ .  
Scope: **local**  
Type: **required**  
An integer value that contains an error code.

## Usage notes

1. If there is no overlap in the data distribution, no operations are performed;
2. The operator  $P^T$  performs the reduction sum of overlap elements; it is a “prolongation” operator  $P^T$  that replicates overlap elements, accounting for the physical replication of data;
3. The operator  $P_a$  performs a scaling on the overlap elements by the amount of replication; thus, when combined with the reduction operator, it implements the average of replicated elements over all of their instances.

## psb\_gather—Gather Global Dense Matrix

These subroutines collect the portions of global dense matrix distributed over all process into one single array stored on one process.

$$glob\_x \leftarrow collect(loc\_x_i)$$

where:

$glob\_x$  is the global submatrix  $glob\_x_{iy:iy+m-1, jy:jy+n-1}$

$loc\_x_i$  is the local portion of global dense matrix on process  $i$ .

$collect$  is the collect function.

$x_i, y$	Subroutine
Long Precision Real	psb_gather
Long Precision Complex	psb_gather

Table 13: Data types

## Syntax

call psb\_gather ( $glob\_x, loc\_x, desc\_a, info, root, iglobx, jglobx, ilocx, jlocx, k$ )

## Syntax

call psb\_gather ( $glob\_x, loc\_x, desc\_a, info, root, iglobx, ilocx$ )

### On Entry

**loc\_x** the local portion of global dense matrix  $glob\_x$ .

Scope: **local**

Type: **required**

Specified as: a rank one or two array containing numbers of the type indicated in Table 13.

**desc\_a** contains data structures for communications.

Scope: **local**

Type: **required**

Specified as: a structured data of type `psb_desc_type`.

**root** The process that holds the global copy. If  $root = -1$  all the processes will have a copy of the global vector.

Scope: **global**

Type: **optional**

Specified as: an integer variable  $0 \leq ix \leq np$ .

**iglobx** Row index to define a submatrix in `glob_x` into which gather the local pieces.

Scope: **global**

Type: **optional**

Specified as: an integer variable  $1 \leq ix \leq \text{matrix\_data}(\text{psb\_m\_})$ .

**jglobx** Column index to define a submatrix in `glob_x` into which gather the local pieces.

Scope: **global**

Type: **optional**

Specified as: an integer variable.

**ilocx** Row index to define a submatrix in `loc_x` that has to be gathered into `glob_x`.

Scope: **local**

Type: **optional**

Specified as: an integer variable.

**jlocx** Columns index to define a submatrix in `loc_x` that has to be gathered into `glob_x`.

Scope: **global**

Type: **optional**

Specified as: an integer variable.

**k** The number of columns to gather.

Scope: **global**

Type: **optional**

Specified as: an integer variable.

### On Return

**glob\_x** The array where the local parts must be gathered.

Scope: **global**

Type: **required**

Specified as: a rank one or two array.

**info** the local portion of result submatrix  $y$ .

Scope: **local**

Type: **required**

An integer value that contains an error code.

## psb\_scatter—Scatter Global Dense Matrix

These subroutines scatters the portions of global dense matrix owned by a process to all the processes in the processes grid.

$$loc\_x_i \leftarrow scatter(glob\_x_i)$$

where:

$glob\_x$  is the global submatrix  $glob\_x_{iy:iy+m-1, jy:jy+n-1}$

$loc\_x_i$  is the local portion of global dense matrix on process  $i$ .

$scatter$  is the scatter function.

$x_i, y$	Subroutine
Long Precision Real	psb_scatter
Long Precision Complex	psb_scatter

Table 14: Data types

## Syntax

call psb\_scatter ( $glob\_x, loc\_x, desc\_a, info, root, iglobx, jglobx, ilocx, jlocx, k$ )

## Syntax

call psb\_scatter ( $glob\_x, loc\_x, desc\_a, info, root, iglobx, ilocx$ )

### On Entry

**glob\_x** The array that must be scattered into local pieces.

Scope: **global**

Type: **required**

Specified as: a rank one or two array.

**desc\_a** contains data structures for communications.

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_desc\\_type](#).

**root** The process that holds the global copy. If  $root = -1$  all the processes have a copy of the global vector.

Scope: **global**

Type: **optional**

Specified as: an integer variable  $0 \leq ix \leq np$ .



- iglobx** Row index to define a submatrix in `glob_x` that has to be scattered into local pieces.  
 Scope: **global**  
 Type: **optional**  
 Specified as: an integer variable  $1 \leq ix \leq matrix\_data(psb\_m\_)$ .
- jglobx** Column index to define a submatrix in `glob_x` that has to be scattered into local pieces.  
 Scope: **global**  
 Type: **optional**  
 Specified as: an integer variable.
- ilocx** Row index to define a submatrix in `loc_x` into which scatter the local piece of `glob_x`.  
 Scope: **local**  
 Type: **optional**  
 Specified as: an integer variable.
- jlocx** Columns index to define a submatrix in `loc_x` into which scatter the local piece of `glob_x`.  
 Scope: **global**  
 Type: **optional**  
 Specified as: an integer variable.
- k** The number of columns to scatter.  
 Scope: **global**  
 Type: **optional**  
 Specified as: an integer variable.

### On Return

- loc\_x** the local portion of global dense matrix `glob_x`.  
 Scope: **local**  
 Type: **required**  
 Specified as: a rank one or two array containing numbers of the type indicated in Table 14.
- info** the local portion of result submatrix `y`.  
 Scope: **local**  
 Type: **required**  
 An integer value that contains an error code.

## **6 Data management, environment handling and auxiliary communication routines**

## **psb\_init**—Initializes PSBLAS parallel environment

### **Syntax**

call `psb_init (ictxt, np)`

This subroutine initializes the PSBLAS parallel environment, defining a virtual parallel machine.

#### **On Entry**

**np** Number of processes in the PSBLAS virtual parallel machine.

Scope:**global**.

Type:**optional**.

Specified as: an integer value. Default: use all available processes provided by the underlying parallel environment.

#### **On Return**

**icontxt** the communication context identifying the virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

### **Notes**

1. A call to this routine must precede any other PSBLAS call.
2. It is an error to specify a value for *np* greater than the number of processes available in the underlying parallel execution environment.

## psb\_info—Return information about PSBLAS parallel environment

### Syntax

call psb\_info (*ictxt*, *iam*, *np*)

This subroutine returns information about the PSBLAS parallel environment, defining a virtual parallel machine.

#### On Entry

**ictxt** the communication context identifying the virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

#### On Return

**iam** Identifier of current process in the PSBLAS virtual parallel machine.

Scope:**local**.

Type:**required**.

Specified as: an integer value.  $-1 \leq iam \leq np - 1$

**np** Number of processes in the PSBLAS virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

### Notes

1. For processes in the virtual parallel machine the identifier will satisfy  $0 \leq iam \leq np - 1$ ;
2. If the user has requested on **psb\_init** a number of processes less than the total available in the parallel execution environment, the remaining processes will have on return  $iam = -1$ ; any such process may only place a call to **psb\_exit**, and is required to do so.

## **psb\_exit—Exit from PSBLAS parallel environment**

### **Syntax**

call psb\_exit (*ictxt*)

This subroutine exits from the PSBLAS parallel virtual machine.

### **On Entry**

**icontxt** the communication context identifying the virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

### **Notes**

1. This routine may be called even if a previous call to **psb\_info** has returned with *iam* = -1; indeed, it is the **ONLY** routine that may be called in this situation, and it is required to do so.

## **psb\_barrier—Synchronization point parallel environment**

### **Syntax**

call psb\_barrier (*ictxt*)

This subroutine acts as a synchronization point for the PSBLAS parallel virtual machine. As such, it must be called by all participating processes.

### **On Entry**

**icontxt** the communication context identifying the virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

## **psb\_abort—Abort a computation**

### **Syntax**

call psb\_abort (*ictxt*)

This subroutine aborts computation on the parallel virtual machine.

### **On Entry**

**icontxt** the communication context identifying the virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

## psb\_cdall—Allocates a communication descriptor

### Syntax

call psb\_cdall (*m*, *n*, *parts*, *icontxt*, *desc\_a*, *info*)

call psb\_cdall (*m*, *v*, *icontxt*, *desc\_a*, *info*, *flag*)

This subroutine initializes the communication descriptor associated with an index space. It takes two forms depending on whether the user specifies the domain partitioning through a subroutine or through a vector

#### First Form: On Entry

**m** the number of rows of the problem.

Scope:**global**.

Type:**required**.

Specified as: an integer value.

**n** the number of columns of the problem.

Scope:**global**.

Type:**required**.

Specified as: an integer value. Currently constrained to be  $m = n$ .

**parts** the subroutine that defines the partitioning scheme.

Scope:**global**.

Type:**required**.

Specified as: a subroutine.

**icontxt** the communication context.

Scope:**global**.

Type:**required**.

Specified as: an integer value.

#### Second Form: On Entry

**m** the size of the index space.

Scope:**global**.

Type:**required**.

Specified as: an integer value  $m > 0$ .

**v** Data allocation: each index  $i \in \{1 \dots m\}$  is allocated to process  $v(i)$ . Scope:**global**.

Type:**required**.

Specified as: an integer array of size  $m$ .

**icontxt** the communication context.

Scope:**global**.

Type:**required**.

Specified as: an integer value.



**flag** Specifies whether entries in  $v$  are zero- or one-based. Scope:**global**.  
Type:**optional**.  
Specified as: an integer value 0, 1, default 0.

#### **On Return**

**desc\_a** the communication descriptor.  
Scope:**local**.  
Type:**required**.  
Specified as: a structured data of type [psb\\_desc\\_type](#).

**info** Error code. Scope: **local**  
Type: **required**  
Specified as: an integer variable.

## **Notes**

1. On exit from this routine the descriptor is in the build state

## psb\_cdins—Communication descriptor insert routine

### Syntax

call psb\_cdins (*nz*, *ia*, *ja*, *desc\_a*, *info*)

#### On Entry

**nz** the number of points being inserted.

Scope: **local**.

Type: **required**.

Specified as: an integer value.

**ia** the row indices of the points being inserted.

Scope: **local**.

Type: **required**.

Specified as: an integer array of length *nz*.

**ja** the column indices of the points being inserted.

Scope: **local**.

Type: **required**.

Specified as: an integer array of length *nz*.

#### On Return

**desc\_a** the updated communication descriptor.

Scope: **local**.

Type: **required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

**info** Error code. Scope: **local**

Type: **required**

Specified as: an integer variable.

### Notes

1. This routine may only be called if the descriptor is in the build state

## psb\_cdasb—Communication descriptor assembly routine

### Syntax

```
call psb_cdasb (desc_a, info)
```

### On Entry

**desc\_a** the communication descriptor.

Scope:**local**.

Type:**required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

### On Return

**info** Error code. Scope: **local**

Type: **required**

Specified as: an integer variable.

### Notes

1. On exit from this routine the descriptor is in the assembled state.

## psb\_cdcpy—Copies a communication descriptor

### Syntax

call psb\_cdcpy (*desc\_out*, *desc\_a*, *info*)

### On Entry

**desc\_a** the communication descriptor.

Scope:**local**.

Type:**required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

### On Return

**desc\_out** the communication descriptor copy.

Scope:**local**.

Type:**required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

**info** Error code. Scope: **local**

Type: **required**

Specified as: an integer variable.

## psb\_cdfree—Frees a communication descriptor

### Syntax

call psb\_cdfree (*desc\_a*, *info*)

### On Entry

**desc\_a** the communication descriptor to be freed.

Scope:**local**.

Type:**required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

### On Return

**info** Error code. Scope: **local**

Type: **required**

Specified as: an integer variable.

## psb\_spall—Allocates a sparse matrix

### Syntax

call psb\_spall (*a*, *desc\_a*, *info*, *nnz*)

#### On Entry

**desc\_a** the communication descriptor.

Scope:**local**.

Type:**required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

**nnz** the number of nonzeros in the local part of the assembled matrix.

Scope: **global**.

Type: **optional**.

Specified as: an integer value.

#### On Return

**a** the matrix to be allocated.

Scope:**local**

Type:**required**

Specified as: a structured data of type [psb\\_spmat\\_type](#).

**info** Error code. Scope: **local**

Type: **required**

Specified as: an integer variable.

### Notes

1. On exit from this routine the sparse matrix is in the build state.
2. The descriptor may be in either the build or assembled state.
3. Providing a good estimate for the number of nonzeros *nnz* in the assembled matrix may substantially improve performance in the matrix build phase, as it will reduce or eliminate the need for multiple data allocations.

## psb\_spins—Insert a cloud of elements into a sparse matrix

### Syntax

call `psb_spins (nz, ia, ja, val, a, desc_a, info)`

#### On Entry

**nz** the number of elements to be inserted.

Scope:**local**.

Type:**required**.

Specified as: an integer scalar.

**ia** the row indices of the elements to be inserted.

Scope:**local**.

Type:**required**.

Specified as: an integer array of size *nz*.

**ja** the column indices of the elements to be inserted.

Scope:**local**.

Type:**required**.

Specified as: an integer array of size *nz*.

**val** the elements to be inserted.

Scope:**local**.

Type:**required**.

Specified as: an array of size *nz*.

**desc\_a** The communication descriptor.

Scope: **local**.

Type: **required**.

Specified as: a variable of type [psb\\_desc\\_type](#).

#### On Return

**a** the matrix into which elements will be inserted.

Scope:**local**

Type:**required**

Specified as: a structured data of type [psb\\_spmat\\_type](#).

**desc\_a** The communication descriptor.

Scope: **local**.

Type: **required**.

Specified as: a variable of type [psb\\_desc\\_type](#).

**info** Error code.  
Scope: **local**  
Type: **required**

## Notes

1. On entry to this routine the descriptor may be in either the build or assembled state.
2. On entry to this routine the sparse matrix may be in either the build or update state.
3. If the descriptor is in the build state, then the sparse matrix ought also be in the build state; the action of the routine is to (implicitly) call `psb_cdins` to add entries to the sparsity pattern;
4. If the descriptor is in the assembled state, then any entries in the sparse matrix that would generate additional communication requirements will be ignored;
5. If the matrix is in the update state, any entries in positions that were not present in the original matrix will be ignored.



## psb\_spasb—Sparse matrix assembly routine

### Syntax

call psb\_spasb (*a*, *desc\_a*, *info*, *afmt*, *upd*, *dupl*)

#### On Entry

**desc\_a** the communication descriptor.

Scope: **local**.

Type: **required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

**afmt** the storage format for the sparse matrix.

Scope: **global**.

Type: **optional**.

Specified as: an array of characters. Default: 'CSR'.

**upd** Provide for updates to the matrix coefficients.

Scope: **global**.

Type: **optional**.

Specified as: integer, possible values: [psb\\_upd\\_srch\\_](#), [psb\\_upd\\_perm\\_](#)

**dupl** How to handle duplicate coefficients.

Scope: **global**.

Type: **optional**.

Specified as: integer, possible values: [psb\\_dupl\\_ovwrt\\_](#), [psb\\_dupl\\_add\\_](#), [psb\\_dupl\\_err\\_](#).

#### On Return

**a** the matrix to be assembled.

Scope: **local**

Type: **required**

Specified as: a structured data of type [psb\\_spmat\\_type](#).

**info** Error code. Scope: **local**

Type: **required**

Specified as: an integer variable.

### Notes

1. On entry to this routine the descriptor must be in the assembled state, i.e. [psb\\_cdasb](#) must already have been called.
2. The sparse matrix may be in either the build or update state;
3. Duplicate entries are detected and handled in both build and update state, with the exception of the error action that is only taken in the build state, i.e. on the first assembly;

4. If the update choice is `psb_upd_perm_`, then subsequent calls to `psb_spins` to update the matrix must be arranged in such a way as to produce exactly the same sequence of coefficient values as encountered at the first assembly;
5. On exit from this routine the matrix is in the assembled state, and thus is suitable for the computational routines.

## psb\_sfree—Frees a sparse matrix

### Syntax

call `psb_sfree (a, desc_a, info)`

#### On Entry

**a** the matrix to be freed.

Scope:**local**

Type:**required**

Specified as: a structured data of type [psb\\_spmat\\_type](#).

**desc\_a** the communication descriptor.

Scope:**local**.

Type:**required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

#### On Return

**info** Error code. Scope: **local**

Type: **required**

Specified as: an integer variable.

## psb\_sprn—Reinit sparse matrix structure for psblas routines.

### Syntax

call psb\_sprn (*a*, *desc\_a*, *info*, *clear*)

#### On Entry

**a** the matrix to be reinitialized.

Scope:**local**

Type:**required**

Specified as: a structured data of type [psb\\_spmat\\_type](#).

**desc\_a** the communication descriptor.

Scope:**local**.

Type:**required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

**clear** Choose whether to zero out matrix coefficients

Scope:**local**.

Type:**optional**.

Default: true.

#### On Return

**info** Error code. Scope: **local**

Type: **required**

Specified as: an integer variable.

### Notes

1. On exit from this routine the sparse matrix is in the update state.

## psb\_geall—Allocates a dense matrix

### Syntax

call psb\_geall (*x*, *desc\_a*, *info*, *n*)

### On Entry

**desc\_a** The communication descriptor.

Scope: **local**

Type: **required**

Specified as: a variable of type [psb\\_desc\\_type](#).

**n** The number of columns of the dense matrix to be allocated.

Scope: **local**

Type: **optional**

Specified as: Integer scalar, default 1. It is ignored if *x* is a rank-1 array.

### On Return

**x** The dense matrix to be allocated.

Scope: **local**

Type: **required**

Specified as: a rank one or two array with the `POINTER` attribute, of type real, complex or integer.

**info** Error code. Scope: **local**

Type: **required**

Specified as: Integer scalar.

## psb\_geins—Dense matrix insertion routine

### Syntax

call psb\_geins (*m*, *irw*, *val*, *x*, *desc\_a*, *info*, *dupl*)

#### On Entry

**m** Number of rows in *val* to be inserted.

Scope: **local**.

Type: **required**.

Specified as: an integer value.

**irw** Indices of the rows to be inserted. Specifically, row *i* of *val* will be inserted into the local row corresponding to the global row index *irw*(*i*).

Scope: **local**.

Type: **required**.

Specified as: an integer array.

**val** the dense submatrix to be inserted.

Scope: **local**.

Type: **required**.

Specified as: a rank 1 or 2 array. Specified as: an integer value.

**desc\_a** the communication descriptor.

Scope: **local**.

Type: **required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

**dupl** How to handle duplicate coefficients.

Scope: **global**.

Type: **optional**.

Specified as: integer, possible values: [psb\\_dupl\\_ovwrt\\_](#), [psb\\_dupl\\_add\\_](#).

#### On Return

**x** the output dense matrix.

Scope: **local**

Type: **required**

Specified as: a rank one or two array with the POINTER attribute, of type real, complex or integer.

**info** Error code. Scope: **local**

Type: **required**

Specified as: an integer variable.

## Notes

1. Dense vectors/matrices do not have an associated state;
2. Duplicate entries are either overwritten or added, there is no provision for raising an error condition.

## psb\_geasb—Assembly a dense matrix

### Syntax

call psb\_geasb (*x*, *desc\_a*, *info*)

#### On Entry

**desc\_a** The communication descriptor.

Scope: **local**

Type: **required**

Specified as: a variable of type [psb.desc.type](#).

#### On Return

**x** The dense matrix to be assembled.

Scope: **local**

Type: **required**

Specified as: a rank one or two array with the POINTER attribute, of type real, complex or integer.

**info** Error code.

Scope: **local**

Type: **required**

Specified as: Integer scalar.



## psb\_gefree—Frees a dense matrix

### Syntax

call `psb_gefree (x, desc_a, info)`

### On Entry

**x** The dense matrix to be freed.

Scope: **local**

Type: **required**

Specified as: a rank one or two array with the `POINTER` attribute, of type real, complex or integer.

**desc\_a** The communication descriptor.

Scope: **local**

Type: **required**

Specified as: a variable of type `psb_desc_type`.

### On Return

**info** Error code.

Scope: **local**

Type: **required**

Specified as: Integer scalar.

## **psb\_gelp**—Applies a left permutation to a dense matrix

### **Syntax**

call `psb_gelp` (*trans*, *iperm*, *x*, *desc\_a*, *info*)

#### **On Entry**

**trans** A character that specifies whether to permute  $A$  or  $A^T$ .  
Scope: **local**  
Type: **required**  
Specified as: a single character with value 'N' for  $A$  or 'T' for  $A^T$ .

**iperm** An integer array containing permutation information.  
Scope: **local**  
Type: **required**  
Specified as: an integer one-dimensional array.

**x** The dense matrix to be permuted.  
Scope: **local**  
Type: **required**  
Specified as: a one or two dimensional array.

**desc\_a** The communication descriptor.  
Scope: **local**  
Type: **required**  
Specified as: a variable of type `psb.desc_type`.

#### **On Return**

**info** Error code.  
Scope: **local**  
Type: **required**  
Specified as: Integer scalar.

## psb\_glob\_to\_loc—Global to local indices conversion

### Syntax

call psb\_glob\_to\_loc (*x*, *y*, *desc\_a*, *info*, *iact*)

call psb\_glob\_to\_loc (*x*, *desc\_a*, *info*, *iact*)

### On Entry

**x** An integer vector of indices to be converted.

Scope: **local**

Type: **required**

Specified as: a rank one integer array.

**desc\_a** the communication descriptor.

Scope: **local**.

Type: **required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

**iact** specifies action to be taken in case of range errors. Scope: **global**

Type: **optional**

Specified as: a character variable E, W or A.

### On Return

**x** If *y* is not present, then *x* is overwritten with the translated integer indices.

Scope: **global**

Type: **required**

Specified as: a rank one integer array.

**y** If *y* is not present, then *y* is overwritten with the translated integer indices, and *x* is left unchanged. Scope: **global**

Type: **optional**

Specified as: a rank one integer array.

**info** Error code. Scope: **local**

Type: **required**

Specified as: an integer variable.

## psb\_loc\_to\_glob—Local to global indices conversion

### Syntax

call psb\_loc\_to\_glob (*x*, *y*, *desc\_a*, *info*, *iact*)

call psb\_loc\_to\_glob (*x*, *desc\_a*, *info*, *iact*)

### On Entry

**x** An integer vector of indices to be converted.

Scope: **local**

Type: **required**

Specified as: a rank one integer array.

**desc\_a** the communication descriptor.

Scope: **local**.

Type: **required**.

Specified as: a structured data of type [psb\\_desc\\_type](#).

**iact** specifies action to be taken in case of range errors. Scope: **global**

Type: **optional**

Specified as: a character variable **E**, **W** or **A**.

### On Return

**x** If *y* is not present, then *x* is overwritten with the translated integer indices.

Scope: **global**

Type: **required**

Specified as: a rank one integer array.

**y** If *y* is not present, then *y* is overwritten with the translated integer indices, and *x* is left unchanged. Scope: **global**

Type: **optional**

Specified as: a rank one integer array.

**info** Error code. Scope: **local**

Type: **required**

Specified as: an integer variable.

## psb\_bcast—Broadcast data

### Syntax

call `psb_bcast (ictxt, dat, root)`

This subroutine implements a broadcast operation based on the underlying communication library.

#### On Entry

**ictxt** the communication context identifying the virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

**dat** On the root process, the data to be broadcast.

Scope:**global**.

Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

**root** Root process holding data to be broadcast.

Scope:**global**.

Type:**required**.

Specified as: an integer value  $0 \leq \text{root} \leq np - 1$ .

#### On Return

**dat** On processes other than root, the data to be broadcast.

Scope:**global**.

Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

## psb\_sum—Global sum

### Syntax

call psb\_sum (*ictxt*, *dat*, *dst*)

This subroutine implements a sum reduction operation based on the underlying communication library.

#### On Entry

**ictxt** the communication context identifying the virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

**dat** The local contribution to the global sum.

Scope:**global**.

Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

**dst** Process to hold the final sum, or  $-1$  to make it available on all processes.

Scope:**global**.

Type:**optional**.

Specified as: an integer value  $-1 \leq dst \leq np - 1$ , default  $-1$ .

#### On Return

**dat** On destination process(es), the result of the sum operation.

Scope:**global**.

Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array.

Type, rank and size must agree on all processes.

## psb\_amx—Global maximum absolute value

### Syntax

call psb\_amx (*ictxt*, *dat*, *dst*)

This subroutine implements a maximum absolute value reduction operation based on the underlying communication library.

#### On Entry

**icontxt** the communication context identifying the virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

**dat** The local contribution to the global maximum.

Scope:**local**.

Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

**dst** Process to hold the final sum, or  $-1$  to make it available on all processes.

Scope:**global**.

Type:**optional**.

Specified as: an integer value  $-1 \leq dst \leq np - 1$ , default  $-1$ .

#### On Return

**dat** On destination process(es), the result of the maximum operation.

Scope:**global**.

Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

## psb\_amn—Global minimum absolute value

### Syntax

call psb\_amn (*ictxt*, *dat*, *dst*)

This subroutine implements a minimum absolute value reduction operation based on the underlying communication library.

#### On Entry

**ictxt** the communication context identifying the virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

**dat** The local contribution to the global minimum.

Scope:**local**.

Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

**dst** Process to hold the final sum, or  $-1$  to make it available on all processes.

Scope:**global**.

Type:**optional**.

Specified as: an integer value  $-1 \leq dst \leq np - 1$ , default  $-1$ .

#### On Return

**dat** On destination process(es), the result of the minimum operation.

Scope:**global**.

Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array.

Type, rank and size must agree on all processes.



## psb\_snd—Send data

### Syntax

call `psb_snd (ictxt, dat, dst, m)`

This subroutine sends a packet of data to a destination.

#### On Entry

**ictxt** the communication context identifying the virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

**dat** The data to be sent.

Scope:**local**.

Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type and rank must agree on sender and receiver process; if  $m$  is not specified, size must agree as well.

**dst** Destination process.

Scope:**global**.

Type:**required**.

Specified as: an integer value  $0 \leq dst \leq np - 1$ .

**m** Number of rows.

Scope:**global**.

Type:**Optional**.

Specified as: an integer value  $0 \leq m \leq size(dat, 1)$ .

When  $dat$  is a rank 2 array, specifies the number of rows to be sent independently of the leading dimension  $size(dat, 1)$ ; must have the same value on sending and receiving processes.

#### On Return

## psb\_rcv—Receive data

### Syntax

call psb\_rcv (*ictxt*, *dat*, *src*, *m*)

This subroutine receives a packet of data to a destination.

#### On Entry

**ictxt** the communication context identifying the virtual parallel machine.

Scope:**global**.

Type:**required**.

Specified as: an integer variable.

**src** Source process.

Scope:**global**.

Type:**required**.

Specified as: an integer value  $0 \leq \textit{src} \leq \textit{np} - 1$ .

**m** Number of rows.

Scope:**global**.

Type:**Optional**.

Specified as: an integer value  $0 \leq m \leq \textit{size}(\textit{dat}, 1)$ .

When *dat* is a rank 2 array, specifies the number of rows to be sent independently of the leading dimension  $\textit{size}(\textit{dat}, 1)$ ; must have the same value on sending and receiving processes.

#### On Return

**dat** The data to be received.

Scope:**local**.

Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type and rank must agree on sender and receiver process; if *m* is not specified, size must agree as well.

## **7 Iterative Methods**

In this chapter we provide routines for preconditioners and iterative methods.

## psb\_cg —CG Iterative Method

This subroutine implements the CG method with restarting. The stopping criterion is the normwise backward error, in the infinity norm, i.e. the iteration is stopped when

$$\frac{\|r\|}{(\|A\|\|x\| + \|b\|)} < eps$$

or

$$\frac{\|r_i\|}{\|b\|_2} < eps$$

according to the value passed through the `istop` argument (see later).

### Syntax

call psb\_cg (*a,prec,b,x,eps,desc\_a,info,itmax,iter,err,itrace,istop*)

#### On Entry

- a** the local portion of global sparse matrix *A*.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type [psb\\_spmat\\_type](#).
- prec** The data structure containing the preconditioner.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type [psb\\_prec\\_type](#).
- b** The RHS vector.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- x** The initial guess.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- eps** The stopping tolerance.  
Scope: **global**  
Type: **required**  
Specified as: a real number.
- desc\_a** contains data structures for communications.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type [psb\\_desc\\_type](#).

**itmax** The maximum number of iterations to perform.

Scope: **global**

Type: **optional**

Default:  $itmax = 1000$ .

Specified as: an integer variable  $itmax \geq 1$ .

**itrace** If  $> 0$  print out an informational message about convergence every *itrace* iterations.

Scope: **global**

Type: **optional**

**istop** An integer specifying the stopping criterion.

Scope: **global**

Type: **optional**

### On Return

**x** The computed solution.

Scope: **local**

Type: **required**

Specified as: a rank one array.

**iter** The number of iterations performed.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

**err** The error estimate on exit.

Scope: **global**

Type: **optional**

Returned as: a real number.

**info** An error code.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

## psb\_cgs — CGS Iterative Method

This subroutine implements the CGS method with restarting. The stopping criterion is the normwise backward error, in the infinity norm, i.e. the iteration is stopped when

$$\frac{\|r\|}{(\|A\|\|x\| + \|b\|)} < eps$$

or

$$\frac{\|r_i\|}{\|b\|_2} < eps$$

according to the value passed through the `istop` argument (see later).

### Syntax

call `psb_cgs (a,prec,b,x,eps,desc_a,info,itmax,iter,err,itrace,istop)`

#### On Entry

- a** the local portion of global sparse matrix  $A$ .  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type `psb_spmat_type`.
- prec** The data structure containing the preconditioner.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type `psb_prec_type`.
- b** The RHS vector.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- x** The initial guess.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- eps** The stopping tolerance.  
Scope: **global**  
Type: **required**  
Specified as: a real number.
- desc\_a** contains data structures for communications.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type `psb_desc_type`.

**itmax** The maximum number of iterations to perform.

Scope: **global**

Type: **optional**

Default:  $itmax = 1000$ .

Specified as: an integer variable  $itmax \geq 1$ .

**itrace** If  $> 0$  print out an informational message about convergence every *itrace* iterations.

Scope: **global**

Type: **optional**

**istop** An integer specifying the stopping criterion.

Scope: **global**

Type: **optional**

### On Return

**x** The computed solution.

Scope: **local**

Type: **required**

Specified as: a rank one array.

**iter** The number of iterations performed.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

**err** The error estimate on exit.

Scope: **global**

Type: **optional**

Returned as: a real number.

**info** An error code.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

## psb\_bicg — BiCG Iterative Method

This subroutine implements the BiCG method with restarting. The stopping criterion is the normwise backward error, in the infinity norm, i.e. the iteration is stopped when

$$\frac{\|r\|}{(\|A\|\|x\| + \|b\|)} < eps$$

or

$$\frac{\|r_i\|}{\|b\|_2} < eps$$

according to the value passed through the `istop` argument (see later).

### Syntax

call `psb_bicg (a,prec,b,x,eps,desc_a,info,itmax,iter,err,itrace,istop)`

#### On Entry

- a** the local portion of global sparse matrix *A*.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type `psb_spmat_type`.
- prec** The data structure containing the preconditioner.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type `psb_prec_type`.
- b** The RHS vector.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- x** The initial guess.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- eps** The stopping tolerance.  
Scope: **global**  
Type: **required**  
Specified as: a real number.
- desc\_a** contains data structures for communications.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type `psb_desc_type`.



**itmax** The maximum number of iterations to perform.

Scope: **global**

Type: **optional**

Default:  $itmax = 1000$ .

Specified as: an integer variable  $itmax \geq 1$ .

**itrace** If  $> 0$  print out an informational message about convergence every *itrace* iterations.

Scope: **global**

Type: **optional**

**istop** An integer specifying the stopping criterion.

Scope: **global**

Type: **optional**

### On Return

**x** The computed solution.

Scope: **local**

Type: **required**

Specified as: a rank one array.

**iter** The number of iterations performed.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

**err** The error estimate on exit.

Scope: **global**

Type: **optional**

Returned as: a real number.

**info** An error code.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

## psb\_bicgstab —BiCGSTAB Iterative Method

This subroutine implements the BiCGSTAB method with restarting. The stopping criterion is the normwise backward error, in the infinity norm, i.e. the iteration is stopped when

$$\frac{\|r\|}{(\|A\|\|x\| + \|b\|)} < eps$$

or

$$\frac{\|r_i\|}{\|b\|_2} < eps$$

according to the value passed through the *istop* argument (see later).

### Syntax

call psb\_bicgstab (*a,prec,b,x,eps,desc\_a,info,itmax,iter,err,itrace,istop*)

#### On Entry

- a** the local portion of global sparse matrix *A*.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type [psb\\_spmat\\_type](#).
- prec** The data structure containing the preconditioner.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type [psb\\_prec\\_type](#).
- b** The RHS vector.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- x** The initial guess.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- eps** The stopping tolerance.  
Scope: **global**  
Type: **required**  
Specified as: a real number.
- desc\_a** contains data structures for communications.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type [psb\\_desc\\_type](#).

**itmax** The maximum number of iterations to perform.

Scope: **global**

Type: **optional**

Default:  $itmax = 1000$ .

Specified as: an integer variable  $itmax \geq 1$ .

**itrace** If  $> 0$  print out an informational message about convergence every *itrace* iterations.

Scope: **global**

Type: **optional**

**istop** An integer specifying the stopping criterion.

Scope: **global**

Type: **optional**

### On Return

**x** The computed solution.

Scope: **local**

Type: **required**

Specified as: a rank one array.

**iter** The number of iterations performed.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

**err** The error estimate on exit.

Scope: **global**

Type: **optional**

Returned as: a real number.

**info** An error code.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

## psb\_bicgstabl — BiCGSTAB-*l* Iterative Method

This subroutine implements the BiCGSTAB-*l* method with restarting. The stopping criterion is the normwise backward error, in the infinity norm, i.e. the iteration is stopped when

$$\frac{\|r\|}{(\|A\|\|x\| + \|b\|)} < eps$$

or

$$\frac{\|r_i\|}{\|b\|_2} < eps$$

according to the value passed through the *istop* argument (see later).

### Syntax

call psb\_bicgstab (*a,prec,b,x,eps,desc\_a,info,itmax,iter,err,itrace,irst,istop*)

#### On Entry

- a** the local portion of global sparse matrix *A*.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type [psb\\_spmat\\_type](#).
- prec** The data structure containing the preconditioner.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type [psb\\_prec\\_type](#).
- b** The RHS vector.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- x** The initial guess.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- eps** The stopping tolerance.  
Scope: **global**  
Type: **required**  
Specified as: a real number.
- desc\_a** contains data structures for communications.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type [psb\\_desc\\_type](#).

**itmax** The maximum number of iterations to perform.

Scope: **global**

Type: **optional**

Default:  $itmax = 1000$ .

Specified as: an integer variable  $itmax \geq 1$ .

**itrace** If  $> 0$  print out an informational message about convergence every *itrace* iterations.

Scope: **global**

Type: **optional**

**irst** An integer specifying the restarting iteration.

Scope: **global**

Type: **optional**

**istop** An integer specifying the stopping criterion.

Scope: **global**

Type: **optional**

### On Return

**x** The computed solution.

Scope: **local**

Type: **required**

Specified as: a rank one array.

**iter** The number of iterations performed.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

**err** The error estimate on exit.

Scope: **global**

Type: **optional**

Returned as: a real number.

**info** An error code.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

## psb\_gmres — GMRES Iterative Method

This subroutine implements the GMRES method with restarting. The stopping criterion is the normwise backward error, in the infinity norm, i.e. the iteration is stopped when

$$\frac{\|r\|}{(\|A\|\|x\| + \|b\|)} < eps$$

or

$$\frac{\|r_i\|}{\|b\|_2} < eps$$

according to the value passed through the `istop` argument (see later).

### Syntax

call `psb_gmres` (*a,prec,b,x,eps,desc\_a,info,itmax,iter,err,itrace,irst,istop*)

#### On Entry

- a** the local portion of global sparse matrix *A*.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type `psb_spmat_type`.
- prec** The data structure containing the preconditioner.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type `psb_prec_type`.
- b** The RHS vector.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- x** The initial guess.  
Scope: **local**  
Type: **required**  
Specified as: a rank one array.
- eps** The stopping tolerance.  
Scope: **global**  
Type: **required**  
Specified as: a real number.
- desc\_a** contains data structures for communications.  
Scope: **local**  
Type: **required**  
Specified as: a structured data of type `psb_desc_type`.

**itmax** The maximum number of iterations to perform.

Scope: **global**

Type: **optional**

Default:  $itmax = 1000$ .

Specified as: an integer variable  $itmax \geq 1$ .

**itrace** If  $> 0$  print out an informational message about convergence every *itrace* iterations.

Scope: **global**

Type: **optional**

**irst** An integer specifying the restart iteration.

Scope: **global**

Type: **optional**

**istop** An integer specifying the stopping criterion.

Scope: **global**

Type: **optional**

### On Return

**x** The computed solution.

Scope: **local**

Type: **required**

Specified as: a rank one array.

**iter** The number of iterations performed.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

**err** The error estimate on exit.

Scope: **global**

Type: **optional**

Returned as: a real number.

**info** An error code.

Scope: **global**

Type: **optional**

Returned as: an integer variable.

## 8 Preconditioner routines

PSBLAS contains the implementation of many preconditioning techniques some of which are very flexible thanks to the presence of many parameters that is possible to adjust to fit the user's needs:

- Diagonal Scaling
- Block Jacobi with ILU(0) factorization
- Additive Schwarz with the Restricted Additive Schwarz and Additive Schwarz with Harmonic extensions;

The PSBLAS library is incorporating a package of two-level Additive Schwarz preconditioners called MD2P4; this is actually a family of preconditioners since there is the possibility to choose between many variants, and is currently in an experimental state. Its documentation is planned to appear after stabilization of the package, which will characterize release 2.1 of our library.



## psb\_precset—Sets the preconditioner type

### Syntax

call psb\_precset (*prec*, *ptype*, *iv*, *rs*, *ierr*)

### On Entry

- prec** Scope: **local**  
Type: **required**  
Specified as: a preconditioner data structure [psb\\_prec\\_type](#).
- ptype** the type of preconditioner. Scope: **global**  
Type: **required**  
Specified as: a character string, see usage notes.
- iv** integer parameters for the preconditioner. Scope: **global**  
Type: **required**  
Specified as: an integer array, see usage notes.
- rs** Scope: **global**  
Type: **optional**  
Specified as: a long precision real number.
- ierr** Scope: **global**  
Type: **required**

### Usage Notes

The PSBLAS 2.0 contains a number of preconditioners, ranging from a simple diagonal scaling to 2-level domain decomposition. These preconditioners may use the SuperLU or the UMFPACK software, if installed; see [13, 12]. Legal inputs to this subroutine are interpreted depending on the *ptype* string as follows<sup>1</sup>:

**NONE** No preconditioning, i.e. the preconditioner is just a copy operator.

**DIAG** Diagonal scaling; each entry of the input vector is multiplied by the reciprocal of the sum of the absolute values of the coefficients in the corresponding row of matrix  $A$ ;

**ILU** Precondition by the incomplete LU factorization of the block-diagonal of matrix  $A$ , where block boundaries are determined by the data allocation boundaries for each process; requires no communication. Only  $ILU(0)$  is currently implemented.

---

<sup>1</sup>The string is case-insensitive

**AS** Additive Schwarz preconditioner (see [6]); in this case the user may specify additional flags through the integer vector `ir` as follows:

- iv*(1) Number of overlap levels, an integer  $novr \geq 0$ , default  $novr = 1$ .
- iv*(2) Restriction operator, legal values: `psb_halo_`, `psb_none_`; default: `psb_halo_`
- iv*(3) Prolongation operator, legal values: `psb_none_`, `psb_sum_`, `psb_avg_`, default: `psb_none_`
- iv*(4) Factorization type, legal values: `f_ilu_n_`, `f_slu_`, `f_umf_`, default: `f_ilu_n_`.

Note that the default corresponds to a Restricted Additive Schwarz preconditioner with  $ILLU(0)$  and 1 level of overlap.

## psb\_precbld—Builds a preconditioner

### Syntax

call `psb_precbld (a, desc_a, prec, info, upd)`

#### On Entry

**a** the system sparse matrix. Scope: **local**

Type: **required**

Specified as: a sparse matrix data structure [psb\\_spmat\\_type](#).

**desc\_a** the problem communication descriptor. Scope: **local**

Type: **required**

Specified as: a communication descriptor data structure [psb\\_desc\\_type](#).

**upd** Scope: **global**

Type: **optional**

Specified as: a character.

#### On Return

**prec** the preconditioner.

Scope: **local**

Type: **required**

Specified as: a preconditioner data structure [psb\\_prec\\_type](#)

**info** the return error code.

Scope: **global**

Type: **required**

Specified as: an integer, upon successful completion  $info = 0$

## psb\_precaply—Preconditioner application routine

### Syntax

call psb\_precaply (*prec,x,y,desc\_a,info,trans,work*)

### Syntax

call psb\_precaply (*prec,x,desc\_a,info,trans*)

### On Entry

**prec** the preconditioner. Scope: **local**

Type: **required**

Specified as: a preconditioner data structure [psb\\_prec\\_type](#).

**x** the source vector. Scope: **local**

Type: **required**

Specified as: a double precision array.

**desc\_a** the problem communication descriptor. Scope: **local**

Type: **required**

Specified as: a communication data structure [psb\\_desc\\_type](#).

**trans** Scope:

Type: **optional**

Specified as: a character.

**work** an optional work space Scope: **local**

Type: **optional**

Specified as: a double precision array.

### On Return

**y** the destination vector. Scope: **local**

Type: **required**

Specified as: a double precision array.

**info** the return error code.

Scope: **local**

Type: **required**

Specified as: an integer, upon successful completion *info* = 0 .

**psb\_prec\_descr**—Prints a description of current preconditioner

## Syntax

call `psb_prec_descr` (*prec*)

## On Entry

**prec** the preconditioner. Scope: **local**

Type: **required**

Specified as: a preconditioner data structure [psb\\_prec\\_type](#).

## 9 Error handling

The PSBLAS library error handling policy has been completely rewritten in version 2.0. The idea behind the design of this new error handling strategy is to keep error messages on a stack allowing the user to trace back up to the point where the first error message has been generated. Every routine in the PSBLAS-2.0 library has, as last non-optional argument, an integer `info` variable; whenever, inside the routine, an error is detected, this variable is set to a value corresponding to a specific error code. Then this error code is also pushed on the error stack and then either control is returned to the caller routine or the execution is aborted, depending on the users choice. At the time when the execution is aborted, an error message is printed on standard output with a level of verbosity than can be chosen by the user. If the execution is not aborted, then, the caller routine checks the value returned in the `info` variable and, if not zero, an error condition is raised. This process continues on all the levels of nested calls until the level where the user decides to abort the program execution.

Figure 5 shows the layout of a generic `psb_foo` routine with respect to the PSBLAS-2.0 error handling policy. It is possible to see how, whenever an error condition is detected, the `info` variable is set to the corresponding error code which is, then, pushed on top of the stack by means of the `psb_errpush`. An error condition may be directly detected inside a routine or indirectly checking the error code returned returned by a called routine. Whenever an error is encountered, after it has been pushed on stack, the program execution skips to a point where the error condition is handled; the error condition is handled either by returning control to the caller routine or by calling the `psb\_error` routine which prints the content of the error stack and aborts the program execution.

Figure 6 reports a sample error message generated by the PSBLAS-2.0 library. This error has been generated by the fact that the user has chosen the invalid “FOO” storage format to represent the sparse matrix. From this error message it is possible to see that the error has been detected inside the `psb_cest` subroutine called by `psb_spasb ...` by process 0 (i.e. the root process).

```

subroutine psb_foo(some args, info)
  ...
  if(error detected) then
    info=errcode1
    call psb_errpush('psb_foo', errcode1)
    goto 9999
  end if
  ...
  call psb_bar(some args, info)
  if(info .ne. zero) then
    info=errcode2
    call psb_errpush('psb_foo', errcode2)
    goto 9999
  end if
  ...
9999 continue
  if (err_act .eq. act_abort) then
    call psb_error(icontxt)
    return
  else
    return
  end if
end subroutine psb_foo

```

Figure 5: The layout of a generic psb\_foo routine with respect to PSBLAS-2.0 error handling policy.

```

=====
Process: 0. PSBLAS Error (4010) in subroutine: df_sample
Error from call to subroutine mat dist
=====
Process: 0. PSBLAS Error (4010) in subroutine: mat_distv
Error from call to subroutine psb_spasb
=====
Process: 0. PSBLAS Error (4010) in subroutine: psb_spasb
Error from call to subroutine psb_cest
=====
Process: 0. PSBLAS Error (136) in subroutine: psb_cest
Format F00 is unknown
=====
Aborting...

```

Figure 6: A sample PSBLAS-2.0 error message. Process 0 detected an error condition inside the psb\_cest subroutine

## **psb\_errpush**—Pushes an error code onto the error stack

### **Syntax**

call psb\_errpush (*err\_c*, *r\_name*, *i\_err*, *a\_err*)

#### **On Entry**

**err\_c** the error code

Scope: **local**

Type: **required**

Specified as: an integer.

**r\_name** the routine where the error has been caught.

Scope: **local**

Type: **required**

Specified as: a string.

**i\_err** additional info for error code

Scope: **local**

Type: **optional**

Specified as: an integer array

**a\_err** additional info for error code

Scope: **local**

Type: **optional**

Specified as: a string.



**psb\_error**—Prints the error stack content and aborts execution

## Syntax

call `psb_error (icontxt)`

## On Entry

**icontxt** the communication context.

Scope: **global**

Type: **optional**

Specified as: an integer.

**psb\_set\_errverbosity**—Sets the verbosity of error messages.

## Syntax

```
call psb_set_errverbosity (v)
```

### On Entry

**v** the verbosity level  
Scope: **global**  
Type: **required**  
Specified as: an integer.

**psb\_set\_erraction**—Set the type of action to be taken upon error condition.

## Syntax

call `psb_set_erraction (err_act)`

## On Entry

**err\_act** the type of action.

Scope: **global**

Type: **required**

Specified as: an integer.

## psb\_errcomm—Error communication routine

### Syntax

call psb\_errcomm (*icontxt*, *err*)

### On Entry

**icontxt** the communication context.

Scope: **global**

Type: **required**

Specified as: an integer.

**err** the error code to be communicated

Scope: **global**

Type: **required**

Specified as: an integer.

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