PSBLAS-2.1 User's guide

A reference guide for the Parallel Sparse BLAS library

by Salvatore Filippone and Alfredo Buttari

"Tor Vergata" University of Rome. April 11, 2007

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1 Introduction

The PSBLAS library, developed with the aim to facilitate the parallelization of computationally intensive scientific applications, is designed to address parallel implementation of iterative solvers for sparse linear systems through the distributed memory paradigm. It includes routines for multiplying sparse matrices by dense matrices, solving block diagonal systems with triangular diagonal entries, preprocessing sparse matrices, and contains additional routines for dense matrix operations. The current implementation of PSBLAS addresses a distributed memory execution model operating with message passing.

The PSBLAS library is internally implemented in a mixture of Fortran 77 and Fortran 95 [21] programming languages. A similar approach has been advocated by a number of authors, e.g. [20]. Moreover, the Fortran 95 facilities for dynamic memory management and interface overloading greatly enhance the usability of the PSBLAS subroutines. In this way, the library can take care of runtime memory requirements that are quite difficult or even impossible to predict at implementation or compilation time. In the current release we rely on the availability of the so-called allocatable extensions, specified in TR 15581. Strictly speaking they are outside the Fortran 95 standard; however they have been included in the Fortran 2003 language standard, and are available in practically all Fortran 95 compilers on the market, including the GNU Fortran compiler from the Free Software Foundation (as of version 4.2). The presentation of the PSBLAS library follows the general structure of the proposal for serial Sparse BLAS [15, 16], which in its turn is based on the proposal for BLAS on dense matrices [1, 2, 3].

The applicability of sparse iterative solvers to many different areas causes some terminology problems because the same concept may be denoted through different names depending on the application area. The PSBLAS features presented in this document will be discussed referring to a finite difference discretization of a Partial Differential Equation (PDE). However, the scope of the library is wider than that: for example, it can be applied to finite element discretizations of PDEs, and even to different classes of problems such as nonlinear optimization, for example in optimal control problems.

The design of a solver for sparse linear systems is driven by many conflicting objectives, such as limiting occupation of storage resources, exploiting regularities in the input data, exploiting hardware characteristics of the parallel platform. To achieve an optimal communication to computation ratio on distributed memory machines it is essential to keep the *data locality* as high as possible; this can be done through an appropriate data allocation strategy. The choice of the preconditioner is another very important factor that affects efficiency of the implemented application. Optimal data distribution requirements for a given preconditioner may conflict with distribution requirements of the rest of the solver. Finding the optimal trade-off may be very difficult because it is application dependent. Possible solutions to these problems and other important inputs to the development of the PSBLAS software package have come from an established experience in applying the PSBLAS solvers to computational fluid dynamics applications.

2 General overview

The PSBLAS library is designed to handle the implementation of iterative solvers for sparse linear systems on distributed memory parallel computers. The system coefficient matrix A must be square; it may be real or complex, nonsymmetric, and its sparsity pattern needs not to be symmetric. The serial computation parts are based on the serial sparse BLAS, so that any extension made to the data structures of the serial kernels is available to the parallel version. The overall design and parallelization strategy have been influenced by the structure of the ScaLAPACK parallel library. The layered structure of the PSBLAS library is shown in figure 1; lower layers of the library indicate an encapsulation relationship with upper layers. The ongoing discussion focuses on the Fortran 95 layer immediately below the application layer. The serial parts of the computation on each process are executed through calls to the serial sparse BLAS subroutines. In a similar way, the inter-process message exchanges are implemented through the Basic Linear Algebra Communication Subroutines (BLACS) library [14] that guarantees a portable and efficient communication layer. The Message Passing Interface code is encapsulated within the BLACS layer. However, in some cases, MPI routines are directly used either to improve efficiency or to implement communication patterns for which the BLACS package doesn't provide any method.

In any case we provide wrappers around the BLACS routines so that the user does not need to delve into their details (see Sec. 7).

The type of linear system matrices that we address typically arise in the numerical solution of PDEs; in such a context, it is necessary to pay special attention to the structure of the problem from which the application originates. The nonzero pattern of a matrix arising from the discretization of a PDE is influenced by various factors, such as the shape of the domain, the discretization strategy, and the equation/unknown ordering. The matrix itself can be interpreted as the adjacency matrix of the graph associated with the discretization mesh.

The distribution of the coefficient matrix for the linear system is based on the "owner computes" rule: the variable associated to each mesh point is assigned to a process that will own the corresponding row in the coefficient matrix and will carry out all related computations. This allocation strategy is equivalent to a partition of the discretization mesh into *sub-domains*. Our library supports any distribution that keeps together the coefficients of each matrix row; there are no other constraints on the variable assignment. This choice is consistent with data distributions commonly used in ScaLAPACK such as CYCLIC(N) and BLOCK, as well as completely arbitrary assignments of equation indices to processes. In particular it is consistent with the usage of graph partitioning tools commonly available in the literature, e.g. METIS [19]. Dense vectors conform to sparse matrices, that is, the entries of a vector follow the same distribution of the matrix rows.

We assume that the sparse matrix is built in parallel, where each process generates its own portion. We never require that the entire matrix be available on a single node. However, it is possible to hold the entire matrix in one process and distribute it explicitly¹, even though the resulting bottleneck would make

¹In our prototype implementation we provide sample scatter/gather routines.

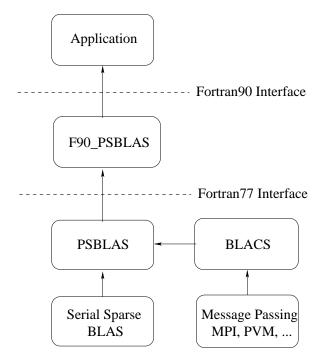


Figure 1: PSBLAS library components hierarchy.

this option unattractive in most cases.

2.1 Basic Nomenclature

Our computational model implies that the data allocation on the parallel distributed memory machine is guided by the structure of the physical model, and specifically by the discretization mesh of the PDE.

Each point of the discretization mesh will have (at least) one associated equation/variable, and therefore one index. We say that point *i* depends on point *j* if the equation for a variable associated with *i* contains a term in *j*, or equivalently if $a_{ij} \neq 0$. After the partition of the discretization mesh into sub-domains assigned to the parallel processes, we classify the points of a given sub-domain as following.

Internal. An internal point of a given domain *depends* only on points of the same domain. If all points of a domain are assigned to one process, then a

computational step (e.g., a matrix-vector product) of the equations associated with the internal points requires no data items from other domains and no communications.

- **Boundary.** A point of a given domain is a boundary point if it *depends* on points belonging to other domains.
- Halo. A halo point for a given domain is a point belonging to another domain such that there is a boundary point which *depends* on it. Whenever performing a computational step, such as a matrix-vector product, the values associated with halo points are requested from other domains. A boundary point of a given domain is a halo point for (at least) another domain; therefore the cardinality of the boundary points set denotes the amount of data sent to other domains.
- **Overlap.** An overlap point is a boundary point assigned to multiple domains. Any operation that involves an overlap point has to be replicated for each assignment.

Overlap points do not usually exist in the basic data distribution, but they are a feature of Domain Decomposition Schwarz preconditioners which we are in the process of including in our distribution [6, 11].

We denote the sets of internal, boundary and halo points for a given subdomain by \mathcal{I} , \mathcal{B} and \mathcal{H} . Each subdomain is assigned to one process; each process usually owns one subdomain, although the user may choose to assign more than one subdomain to a process. If each process *i* owns one subdomain, the number of rows in the local sparse matrix is $|\mathcal{I}_i| + |\mathcal{B}_i|$, and the number of local columns (i.e. those for which there exists at least one non-zero entry in the local rows) is $|\mathcal{I}_i| + |\mathcal{B}_i| + |\mathcal{H}_i|$.

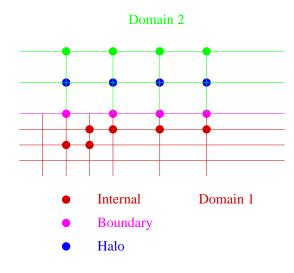


Figure 2: Point classification.

This classification of mesh points guides the naming scheme that we adopted in the library internals and in the data structures. We explicitly note that "Halo" points are also often called "ghost" points in the literature.

2.2 Library contents

The PSBLAS library consists of various classes of subroutines:

Computational routines comprising:

- Sparse matrix by dense matrix product;
- Sparse triangular systems solution for block diagonal matrices;
- Vector and matrix norms;
- Dense matrix sums;
- Dot products.

Communication routines handling halo and overlap communications;

Data management and auxiliary routines including:

- Parallel environment management
- Communication descriptors allocation;
- Dense and sparse matrix allocation;
- Dense and sparse matrix build and update;
- Sparse matrix and data distribution preprocessing.

Preconditioner routines

Iterative methods a subset of Krylov subspace iterative methods

The following naming scheme has been adopted for all the symbols internally defined in the PSBLAS software package:

- all the symbols (i.e. subroutine names, data types...) are prefixed by psb_
- all the data type names are suffixed by _type
- all the constant values are suffixed by _
- all the subroutine names follow the rule psb_xxname where xx can be either:
 - ge: the routine is related to dense data,
 - sp: the routine is related to sparse data,
 - cd: the routine is related to communication descriptor (see 3).

For example the psb_geins, psb_spins and psb_cdins perform the same action (see 6) on dense matrices, sparse matrices and communication descriptors respectively. Interface overloading allows the usage of the same subroutine interfaces for both real and complex data.

In the description of the subroutines, arguments or argument entries are classified as:

global For input arguments, the value must be the same on all processes participating in the subroutine call; for output arguments the value is guaranteed to be the same.

local Each process has its own value(s) independently.

2.3 Application structure

The main underlying principle of the PSBLAS library is that the library objects are created and exist with reference to a discretized space to which there corresponds an index space and a matrix sparsity pattern. As an example, consider a cell-centered finite-volume discretization of the Navier-Stokes equations on a simulation domain; the index space $1 \dots n$ is isomorphic to the set of cell centers, whereas the pattern of the associated linear system matrix is isomorphic to the adjacency graph imposed on the discretization mesh by the discretization stencil.

Thus the first order of business is to establish an index space, and this is done with a call to psb_cdall in which we specify the size of the index space n and the allocation of the elements of the index space to the various processes making up the MPI (virtual) parallel machine.

The index space is partitioned among processes, and this creates a mapping from the "global" numbering $1 \dots n$ to a numbering "local" to each process; each process *i* will own a certain subset $1 \dots n_{\text{roW}_i}$, each element of which corresponds to a certain element of $1 \dots n$. The user does not set explicitly this mapping; when the application needs to indicate to which element of the index space a certain item is related, such as the row and column index of a matrix coefficient, it does so in the "global" numbering, and the library will translate into the appropriate "local" numbering.

For a given index space $1 \dots n$ there are many possible associated topologies, i.e. many different discretization stencils; thus the description of the index space is not completed until the user has defined a sparsity pattern, either explicitly through psb_cdins or implicitly through psb_spins. The descriptor is finalized with a call to psb_cdasb and a sparse matrix with a call to psb_spasb. After psb_cdasb each process *i* will have defined a set of "halo" (or "ghost") indices $n_{\text{row}_i} + 1 \dots n_{\text{col}_i}$, denoting elements of the index space that are *not* assigned to process *i*; however the variables associated with them are needed to complete computations associated with the sparse matrix *A*, and thus they have to be fetched from (neighbouring) processes. The descriptor of the index space is built exactly for the purpose of properly sequencing the communication steps required to achieve this objective.

A simple application structure will walk through the index space allocation, matrix/vector creation and linear system solution as follows:

- 1. Initialize parallel environment with psb_init
- 2. Initialize index space with psb_cdall
- 3. Allocate sparse matrix and dense vectors with psb_spall and psb_geall
- 4. Loop over all local rows, generate matrix and vector entries, and insert them with psb_spins and psb_geins
- 5. Assemble the various entities:
 - (a) psb_cdasb
 - (b) psb_spasb
 - (c) psb_geasb

- 6. Choose the preconditioner to be used with psb_precset and build it with psb_precbld
- 7. Call the iterative method of choice, e.g. psb_bicgstab

This is the structure of the sample program test/pargen/ppde90.f90.

For a simulation in which the same discretization mesh is used over multiple time steps, the following structure may be more appropriate:

- 1. Initialize parallel environment with psb_init
- 2. Initialize index space with psb_cdall
- 3. Loop over the topology of the discretization mesh and build the descriptor with psb_cdins
- 4. Assemble the descriptor with psb_cdasb
- 5. Allocate the sparse matrices and dense vectors with psb_spall and psb_geall
- 6. Loop over the time steps:
 - (a) If after first time step, reinitialize the sparse matrix with psb_sprn; also zero out the dense vectors;
 - (b) Loop over the mesh, generate the coefficients and insert/update them with psb_spins and psb_geins
 - (c) Assemble with psb_spasb and psb_geasb
 - (d) Choose and build preconditioner with psb_precset and psb_precbld
 - (e) Call the iterative method of choice, e.g. psb_bicgstab

The insertion routines will be called as many times as needed; they only need to be called on the data that is actually allocated to the current process, i.e. each process generates its own data.

In principle there is no specific order in the calls to psb_spins, nor is there a requirement to build a matrix row in its entirety before calling the routine; this allows the application programmer to walk through the discretization mesh element by element, generating the main part of a given matrix row but also contributions to the rows corresponding to neighbouring elements.

From a functional point of view it is even possible to execute one call for each nonzero coefficient; however this would have a substantial computational overhead. It is therefore advisable to pack a certain amount of data into each call to the insertion routine, say touching on a few tens of rows; the best performing value would depend on both the architecture of the computer being used and on the problem structure. At the opposite extreme, it would be possible to generate the entire part of a coefficient matrix residing on a process and pass it in a single call to psb_spins; this, however, would entail a doubling of memory occupation, and thus would be almost always far from optimal.

2.4 Programming model

The PSBLAS librarary is based on the Single Program Multiple Data (SPMD) programming model: each process participating in the computation performs the same actions on a chunk of data. Parallelism is thus data-driven.

Because of this structure, practically all subroutines *must* be called simultaneously by all processes participating in the computation, i.e each subroutine call acts implicitly as a synchronization point. The exceptions to this rule are:

- The insertion routines psb_cdins, psb_spins and psb_geins;
- The error handling routines.

In particular, as per the discussion in the previous section, the insertion routines may be called a different number of times on each process, depending on the data distribution chosen by the user.

3 Data Structures

In this chapter we illustrate the data structures used for definition of routines interfaces. They include data structures for sparse matrices, communication descriptors and preconditioners.

All the data types and the basic subroutine interfaces are defined in the module psb_base_mod; this will have to be included by every user subroutine that makes use of the library.

3.1 Descriptor data structure

All the general matrix informations and elements to be exchanged among processes are stored within a data structure of the type psb_desc_type. Every structure of this type is associated to a sparse matrix, it contains data about general matrix informations and elements to be exchanged among processes.

It is not necessary for the user to know the internal structure of psb_desc_type, it is set in a transparent mode by the tools routines of Sec. 6, and its fields may be accessed if necessary via the routines of sec. 3.4; nevertheless we include a description for the curious reader:

- matrix_data includes general information about matrix and process grid, such as the communication context, the size of the global matrix, the size of the portion of matrix stored on the current process, and so on. Specified as: an allocatable integer array of dimension psb_mdata_size_.
- halo_index A list of the halo and boundary elements for the current process to be exchanged with other processes; for each processes with which it is necessary to communicate:
 - 1. Process identifier;
 - 2. Number of points to be received;
 - 3. Indices of points to be received;
 - 4. Number of points to be sent;
 - 5. Indices of points to be sent;

The list may contain an arbitrary number of groups; its end is marked by a -1.

Specified as: an allocatable integer array of rank one.

- **ext_index** A list of element indices to be exchanged to implement the mapping between a base descriptor and a descriptor with overlap.
- **ovrlap_index** A list of the overlap elements for the current process, organized in groups like the previous vector:
 - 1. Process identifier;
 - 2. Number of points to be received;
 - 3. Indices of points to be received;
 - 4. Number of points to be sent;
 - 5. Indices of points to be sent;

The list may contain an arbitrary number of groups; its end is marked by a -1.

Specified as: an allocatable integer array of rank one.

ovrlap_elem For all overlap points belonging to the current process:

- 1. Overlap point index;
- 2. Number of processes sharing that overlap points;

The list may contain an arbitrary number of groups; its end is marked by a -1.

Specified as: an allocatable integer array of rank one.

loc_to_glob each element i of this array contains global identifier of the local variable i.

Specified as: an allocatable integer array of rank one.

glob_to_loc, glb_lc, hashv Contain a mapping from global to local indices. The mapping may be stored in two different formats depending on the size of the index space.

The Fortran95 definition for psb_desc_type structures is as follows:

```
type psb_desc_type
    integer, allocatable :: matrix_data(:), halo_index(:)
    integer, allocatable :: ext_index(:)
    integer, allocatable :: overlap_elem(:), overlap_index(:)
    integer, allocatable :: loc_to_glob(:), glob_to_loc(:)
    integer, allocatable :: hashv(:), glb_lc(:,:)
end type psb_desc_type
```

Figure 3: The PSBLAS defined data type that contains the communication descriptor.

A communication descriptor associated with a sparse matrix has a state, which can take the following values:

- **Build:** State entered after the first allocation, and before the first assembly; in this state it is possible to add communication requirements among different processes.
- Assembled: State entered after the assembly; computations using the associated sparse matrix, such as matrix-vector products, are only possible in this state.

3.1.1 Named Constants

psb_none_ Generic no-op;

psb_nohalo_ Do not fetch halo elements;

psb_halo_ Fetch halo elements from neighbouring processes;

 psb_sum_- Sum overlapped elements

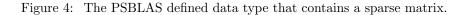
psb_avg_ Average overlapped elements

3.2 Sparse Matrix data structure

The psb_spmat_type data structure contains all information about local portion of the sparse matrix and its storage mode. Most of these fields are set by the tools routines when inserting a new sparse matrix; the user needs only choose, if he/she so whishes, a specific matrix storage mode.

- **aspk** Contains values of the local distributed sparse matrix. Specified as: an allocatable array of rank one of type corresponding to matrix entries type.
- ia1 Holds integer information on distributed sparse matrix. Actual information will depend on data format used.Specified as: an allocatable integer array of rank one.
- ia2 Holds integer information on distributed sparse matrix. Actual information will depend on data format used.Specified as: an allocatable integer array of rank one.
- infoa On entry can hold auxiliary information on distributed sparse matrix. Actual information will depend on data format used. Specified as: an integer array of length psb_ifasize_.
- fida Defines the format of the distributed sparse matrix. Specified as: a string of length 5
- **descra** Describe the characteristic of the distributed sparse matrix. Specified as: array of character of length 9.
- pl Specifies the local row permutation of distributed sparse matrix. If pl(1) is equal to 0, then there isn't row permutation. Specified as: an allocatable integer array of dimension equal to number of local row (matrix_data[psb_n_row_])
- **pr** Specifies the local column permutation of distributed sparse matrix. If PR(1) is equal to 0, then there isn't columnm permutation. Specified as: an allocatable integer array of dimension equal to number of local row (matrix_data[psb_n_col_])
- **m** Number of rows; if row indices are stored explicitly, as in Coordinate Storage, should be greater than or equal to the maximum row index actually present in the sparse matrix. Specified as: integer variable.
- **k** Number of columns; if column indices are stored explicitly, as in Coordinate Storage or Compressed Sparse Rows, should be greater than or equal to the maximum column index actually present in the sparse matrix. Specified as: integer variable.

```
type psb_dspmat_type
    integer          :: m, k
    character          :: fida(5)
    character          :: descra(10)
    integer          :: infoa(psb_ifa_size_)
    real(kind(1.d0)), allocatable :: aspk(:)
    integer, allocatable :: ia1(:), ia2(:)
    integer, allocatable :: pr(:), pl(:)
end type psb_dspmat_type
```



FORTRAN95 interface for distributed sparse matrices containing double precision real entries is defined as in figure 4.

The following two cases are among the most commonly used:

- fida="CSR" Compressed storage by rows. In this case the following should hold:
 - 1. ia2(i) contains the index of the first element of row i; the last element of the sparse matrix is thus stored at index ia2(m+1)-1. It should contain m+1 entries in nondecreasing order (strictly increasing, if there are no empty rows).
 - 2. ia1(j) contains the column index and aspk(j) contains the corresponding coefficient value, for all $ia2(1) \le j \le ia2(m+1) 1$.
- fida="COO" Coordinate storage. In this case the following should hold:
 - 1. infoa(1) contains the number of nonzero elements in the matrix;
 - For all 1 ≤ j ≤ infoa(1), the coefficient, row index and column index are stored into apsk(j), ia1(j) and ia2(j) respectively.
- A sparse matrix has an associated state, which can take the following values:
- **Build:** State entered after the first allocation, and before the first assembly; in this state it is possible to add nonzero entries.
- Assembled: State entered after the assembly; computations using the sparse matrix, such as matrix-vector products, are only possible in this state;
- **Update:** State entered after a reinitalization; this is used to handle applications in which the same sparsity pattern is used multiple times with different coefficients. In this state it is only possible to enter coefficients for already existing nonzero entries.

3.2.1 Named Constants

psb_dupl_ovwrt_ Duplicate coefficients should be overwritten (i.e. ignore duplications) psb_dupl_add_ Duplicate coefficients should be added;

psb_dupl_err_ Duplicate coefficients should trigger an error conditino

psb_upd_dflt_ Default update strategy for matrix coefficients;

psb_upd_srch_ Update strategy based on search into the data structure;

psb_upd_perm_ Update strategy based on additional permutation data (see tools routine description).

3.3 Preconditioner data structure

Our base library offers support for simple well known preconditioners like Diagonal Scaling or Block Jacobi with incomplete factorization ILU(0).

A preconditioner is held in the psb_prec_type data structure reported in figure 5. The psb_prec_type data type may contain a simple preconditioning matrix with the associated communication descriptor. The values contained in the iprcparm and dprcparm define tha type of preconditioner along with all the parameters related to it; thus, iprcparm and dprcparm define how the other records have to be interpreted. This data structure is the basis of ore complex preconditioning strategies, which are the subject of further research.

```
type psb_dprec_type
type(psb_dspmat_type), allocatable :: av(:)
real(kind(1.d0)), allocatable :: d(:)
type(psb_desc_type) :: desc_data
integer, allocatable :: iprcparm(:)
real(kind(1.d0)), allocatable :: dprcparm(:)
integer, allocatable :: perm(:), invperm(:)
integer :: prec, base_prec
end type psb_dprec_type
```

Figure 5: The PSBLAS defined data type that contains a preconditioner.

3.4 Data structure query routines

psb_cd_get_local_rows—Get number of local rows

Syntax

 $nr = psb_cd_get_local_rows$ (desc)

On Entry

desc the communication descriptor.

Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

On Return

Function value The number of local rows, i.e. the number of rows owned by the current process; as explained in 1, it is equal to $|\mathcal{I}_i| + |\mathcal{B}_i|$. The returned value is specific to the calling process.

psb_cd_get_local_cols—Get number of local cols

Syntax

 $nc = psb_cd_get_local_cols (desc)$

On Entry

desc the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

On Return

Function value The number of local cols, i.e. the number of indices used by the current process, including both local and halo indices; as explained in 1, it is equal to $|\mathcal{I}_i| + |\mathcal{B}_i| + |\mathcal{H}_i|$. The returned value is specific to the calling process.

psb_cd_get_global_rows—Get number of global rows

Syntax

 $nr = psb_cd_get_global_rows$ (*desc*)

On Entry

desc the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

On Return

Function value The number of global rows in the mesh

$psb_cd_get_global_cols$ —Get number of global cols

Syntax

 $nr = psb_cd_get_global_cols (desc)$

On Entry

desc the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

On Return

Function value The number of global cols in the mesh

$psb_cd_get_context\\ --Get\ communication\ context\\$

Syntax

 $ictxt = psb_cd_get_context (desc)$

On Entry

desc the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

On Return

Function value The communication context.

psb_sp_get_nrows—Get number of rows in a sparse matrix

Syntax

 $nr = psb_sp_get_nrows (a)$

On Entry

a the sparse matrix Scope:local Type:required Specified as: a structured data of type psb_spmat_type.

On Return

Function value The number of rows of sparse matrix a.

psb_sp_get_ncols—Get number of columns in a sparse matrix

Syntax

 $nr = psb_sp_get_ncols(a)$

On Entry

a the sparse matrix Scope:local Type:required Specified as: a structured data of type psb_spmat_type.

On Return

Function value The number of columns of sparse matrix a.

psb_sp_get_nnzeros—Get number of nonzero elements in a sparse matrix

Syntax

 $nr = psb_sp_get_nnzeros(a)$

On Entry

a the sparse matrix Scope:local Type:required Specified as: a structured data of type psb_spmat_type.

On Return

Function value The number of nonzero elements stored in sparse matrix a.

Notes

1. The function value is specific to the storage format of matrix **a**; some storage formats employ padding, thus the returned value for the same matrix may be different for different storage choices.

4 Computational routines

psb_geaxpby—General Dense Matrix Sum

This subroutine is an interface to the computational kernel for dense matrix sum:

 $y \leftarrow \alpha \, x + \beta y$

Syntax

call psb_geaxpby (alpha, x, beta, y, desc_a, info)

x, y, α, β	Subroutine
Long Precision Real	psb_geaxpby
Long Precision Complex	$psb_geaxpby$

Table 1: Data types

On Entry

alpha the scalar α . Scope: **global** Type: **required** Specified as: a number of the data type indicated in Table 1.

x the local portion of global dense matrix x.
Scope: local
Type: required
Specified as: a rank one or two array containing numbers of type specified in Table 1. The rank of x must be the same of y.

beta the scalar β . Scope: global Type: required Specified as: a number of the data type indicated in Table 1.

y the local portion of the global dense matrix y.
Scope: local
Type: required
Specified as: a rank one or two array containing numbers of the type indicated in Table 1. The rank of y must be the same of x.

desc_a contains data structures for communications. Scope: local Type: required Specified as: a structured data of type psb_desc_type.

On Return

 \mathbf{y} the local portion of result submatrix y.

Scope: **local** Type: **required** Specified as: a rank one or two array containing numbers of the type indicated in Table 1.

info Error code.

psb_gedot—Dot Product

This function computes dot product between two vectors x and y. If x and y are double precision real vectors computes dot-product as:

 $dot \leftarrow x^T y$

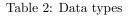
Else if x and y are double precision complex vectors then computes dot-product as:

 $dot \leftarrow x^H y$

Syntax

 $psb_gedot (x, y, desc_a, info)$

dot, x, y	Function
Long Precision Real	psb_gedot
Long Precision Complex	psb_gedot



On Entry

 \mathbf{x} the local portion of global dense matrix x. Scope: **local** Type: **required** Specified as: an array of rank one or two containing numbers of type specified in Table 2. The rank of x must be the same of y.

 \mathbf{y} the local portion of global dense matrix y.

Scope: local Type: required Specified as: an array of rank one or two containing numbers of type specified in Table 2. The rank of y must be the same of x.

desc_a contains data structures for communications. Scope: local Type: required Specified as: a structured data of type psb_desc_type.

On Return

Function value is the dot product of subvectors x and y. Scope: **global** Specified as: a number of the data type indicated in Table 2. info Error code.

psb_gedots—Generalized Dot Product

This subroutine computes a series of dot products among the columns of two dense matrices x and y:

 $res(i) \leftarrow x(:,i)^T y(:,i)$

If the matrices are complex, then the usual convention applies, i.e. the conjugate transpose of x is used. If x and y are of rank one, then res is a scalar, else it is a rank one array.

Syntax

call psb_gedots (res, x, y, desc_a, info)

res, x, y	Subroutine
Long Precision Real	psb_gedots
Long Precision Complex	psb_gedots

Table 3: Data types

On Entry

x the local portion of global dense matrix x. Scope: **local** Type: **required** Specified as: an array of rank one or two containing numbers of type specified in Table 3. The rank of x must be the same of y.

 \mathbf{y} the local portion of global dense matrix y.

Scope: local Type: required Specified as: an array of rank one or two containing numbers of type specified in Table 3. The rank of y must be the same of x.

desc_a contains data structures for communications. Scope: local Type: required Specified as: a structured data of type psb_desc_type.

On Return

res is the dot product of subvectors x and y. Scope: **global** Specified as: a number or a rank-one array of the data type indicated in Table 2. info Error code.

psb_geamax—Infinity-Norm of Vector

This function computes the infinity-norm of a vector x. If x is a double precision real vector computes infinity norm as:

 $amax \leftarrow \max_i |x_i|$

else if x is a double precision complex vector then computes infinity-norm as:

 $amax \leftarrow \max_i \left(|re(x_i)| + |im(x_i)| \right)$

Syntax

 psb_geamax (x, $desc_a$, info)

amax	x	Function
Long Precision Real	Long Precision Real	psb_geamax
Long Precision Real	Long Precision Complex	psb_geamax

Table 4: Data types

On Entry

 \mathbf{x} the local portion of global dense matrix x. Scope: **local** Type: **required** Specified as: a rank one or two array containing numbers of type specified in Table 4.

desc_a contains data structures for communications.
 Scope: local
 Type: required
 Specified as: a structured data of type psb_desc_type.

On Return

Function value is the infinity norm of subvector x.Scope: globalSpecified as: a long precision real number.info Error code.

psb_geamaxs—Generalized Infinity Norm

This subroutine computes a series of infinity norms on the columns of a dense matrix x:

 $res(i) \gets \max_k |x(k,i)|$

Syntax

call psb_geamaxs (res, x, desc_a, info)

res	x	Subroutine
Long Precision Real	Long Precision Real	psb_geamaxs
Long Precision Real	Long Precision Complex	$psb_geamaxs$

Table 5: Data types

On Entry

 \mathbf{x} the local portion of global dense matrix x.

Scope: **local** Type: **required** Specified as: a rank one or two array containing numbers of type specified in Table 5.

 ${\bf desc_a}$ contains data structures for communications.

Scope: local Type: required Specified as: a structured data of type psb_desc_type.

On Return

res is the infinity norm of the columns of x.

Scope: **global** Specified as: a number or a rank-one array of long precision real numbers.

info Error code.

psb_geasum—1-Norm of Vector

This function computes the 1-norm of a vector x. If x is a double precision real vector computes 1-norm as:

 $asum \leftarrow ||x_i||$

else if x is double precision complex vector then computes 1-norm as:

 $asum \leftarrow \|re(x)\|_1 + \|im(x)\|_1$

Syntax

 $psb_geasum (x, desc_a, info)$

asum	x	Function
Long Precision Real	Long Precision Real	psb_geasum
Long Precision Real	Long Precision Complex	psb_geasum

Table 6: Data types

On Entry

 \mathbf{x} the local portion of global dense matrix x. Scope: **local** Type: **required** Specified as: a rank one or two array containing numbers of type specified in Table 6.

desc_a contains data structures for communications.
 Scope: local
 Type: required
 Specified as: a structured data of type psb_desc_type.

On Return

Function value is the 1-norm of vector x.Scope: globalSpecified as: a long precision real number.

psb_geasums—Generalized 1-Norm of Vector

This subroutine computes a series of 1-norms on the columns of a dense matrix x:

 $res(i) \leftarrow \max_k |x(k,i)|$

This function computes the 1-norm of a vector x. If x is a double precision real vector computes 1-norm as:

 $res(i) \leftarrow ||x_i||$

else if x is double precision complex vector then computes 1-norm as:

 $res(i) \leftarrow ||re(x)||_1 + ||im(x)||_1$

Syntax

call psb_geasums (res, x, desc_a, info)

res	x	Subroutine
Long Precision Real	Long Precision Real	psb_geasums
Long Precision Real	Long Precision Complex	$psb_geasums$

Table 7: Data types

On Entry

x the local portion of global dense matrix x.
 Scope: local
 Type: required
 Specified as: a rank one or two array containing numbers of type specified in Table 7.

 $\mathbf{desc_a}$ contains data structures for communications.

Scope: **local** Type: **required**

Specified as: a structured data of type psb_desc_type.

On Return

res contains the 1-norm of (the columns of) x. Scope: **global** Specified as: a long precision real number.

psb_genrm2—2-Norm of Vector

This function computes the 2-norm of a vector x. If x is a double precision real vector computes 2-norm as:

$$nrm2 \leftarrow \sqrt{x^T x}$$

else if x is double precision complex vector then computes 2-norm as:

$$nrm2 \leftarrow \sqrt{x^H x}$$

nrm2	x	Function
Long Precision Real	Long Precision Real	psb_genrm2
Long Precision Real	Long Precision Complex	psb_genrm2

Table 8: Data types

Syntax

 psb_genrm2 (x, $desc_a$, info)

On Entry

 \mathbf{x} the local portion of global dense matrix x. Scope: **local** Type: **required** Specified as: a rank one or two array containing numbers of type specified in Table 8.

desc_a contains data structures for communications. Scope: local Type: required Specified as: a structured data of type psb_desc_type.

On Return

Function Value is the 2-norm of subvector x. Scope: global Type: required Specified as: a long precision real number.

psb_genrm2s—Generalized 1-Norm of Vector

This subroutine computes a series of 1-norms on the columns of a dense matrix x:

$$res(i) \leftarrow \max_k |x(k,i)|$$

This function computes the 1-norm of a vector x. If x is a double precision real vector computes 1-norm as:

$$res(i) \leftarrow \sqrt{x^T x}$$

else if x is double precision complex vector then computes 1-norm as:

$$res(i) \leftarrow \sqrt{x^H x}$$

Syntax

call psb_genrm2s (res, x, desc_a, info)

res	x	Subroutine
Long Precision Real	Long Precision Real	psb_genrm2s
Long Precision Real	Long Precision Complex	$psb_genrm2s$

Table 9: Data types

On Entry

x the local portion of global dense matrix x.
 Scope: local
 Type: required
 Specified as: a rank one or two array containing numbers of type specified in Table 9.

 ${\bf desc_a}$ contains data structures for communications.

Scope: **local** Type: **required**

Specified as: a structured data of type psb_desc_type.

On Return

res contains the 1-norm of (the columns of) x. Scope: **global** Specified as: a long precision real number.

psb_spnrmi—Infinity Norm of Sparse Matrix

This function computes the infinity-norm of a matrix A:

 $nrmi \leftarrow \|A\|_{\infty}$

where:

 ${\cal A}$ represents the global matrix ${\cal A}$

A	Function
Long Precision Real	psb_spnrmi
Long Precision Complex	psb_spnrmi

Table 10: Data types

Syntax

psb_spnrmi (A, desc_a, info)

On Entry

a the local portion of the global sparse matrix A.
Scope: local
Type: required
Specified as: a structured data of type psb_spmat_type.

desc_a contains data structures for communications. Scope: local Type: required Specified as: a structured data of type psb_desc_type.

On Return

Function value is the infinity-norm of sparse submatrix A. Scope: **global** Specified as: a long precision real number.

psb_spmm—Sparse Matrix by Dense Matrix Product

This subroutine computes the Sparse Matrix by Dense Matrix Product:

$$y \leftarrow \alpha P_r A P_c x + \beta y \tag{1}$$

$$y \leftarrow \alpha P_r A^T P_c x + \beta y \tag{2}$$

$$y \leftarrow \alpha P_r A^H P_c x + \beta y \tag{3}$$

where:

x is the global dense submatrix $x_{i,i}$

y is the global dense submatrix $y_{:,:}$

A is the global sparse submatrix A

 ${\cal P}_r, {\cal P}_c$ are the permutation matrices.

A, x, y, α, β	Subroutine
Long Precision Real	psb_spmm
Long Precision Complex	psb_spmm

Table 11: Data types

Syntax

call psb_spmm (alpha, a, x, beta, y, desc_a, info)

call psb_spmm (alpha, a, x, beta, y,desc_a, info, trans, work)

On Entry

alpha the scalar α.
Scope: global
Type: required
Specified as: a number of the data type indicated in Table 11.
a the local portion of the sparse matrix A.

Scope: local Type: required Specified as: a structured data of type psb_spmat_type. x the local portion of global dense matrix x. Scope: local

Type: **required** Specified as: a rank one or two array containing numbers of type specified in Table 11. The rank of x must be the same of y. beta the scalar β . Scope: global Type: required Specified as: a number of the data type indicated in Table 11.

y the local portion of global dense matrix y.
Scope: local
Type: required
Specified as: a rank one or two array containing numbers of type specified in Table 11. The rank of y must be the same of x.

desc_a contains data structures for communications.
 Scope: local
 Type: required
 Specified as: a structured data of type psb_desc_type.

trans indicate what kind of operation to perform.

trans = N the operation is specified by equation 1 trans = T the operation is specified by equation 2 trans = C the operation is specified by equation 3

Scope: global Type: optional Default: trans = NSpecified as: a character variable.

work work array. Scope: local Type: optional Specified as: a rank one array of the same type of x and y with the TARGET attribute.

On Return

 ${f y}$ the local portion of result submatrix y. Scope: local Type: required Specified as: an array of rank one or two containing numbers of type specified in Table 11.

psb_spsm—Triangular System Solve

This subroutine computes the Triangular System Solve:

$$y \leftarrow \alpha P_r T^{-1} P_c x + \beta y$$

$$y \leftarrow \alpha D P_r T^{-1} P_c x + \beta y$$

$$y \leftarrow \alpha P_r T^{-1} P_c D x + \beta y$$

$$y \leftarrow \alpha P_r T^{-T} P_c x + \beta y$$

$$y \leftarrow \alpha D P_r T^{-T} P_c x + \beta y$$

$$y \leftarrow \alpha P_r T^{-T} P_c D x + \beta y$$

$$y \leftarrow \alpha D P_r T^{-H} P_c x + \beta y$$

$$y \leftarrow \alpha P_r T^{-H} P_c x + \beta y$$

where:

- x is the global dense submatrix $x_{:,:}$
- y is the global dense submatrix $y_{:,:}$
- $T\,$ is the global sparse block triangular submatrix $T\,$
- ${\cal D}\,$ is the scaling diagonal matrix.
- ${\cal P}_r, {\cal P}_c\,$ are the permutation matrices.

Syntax

call psb_spsm (alpha, t, x, beta, y, desc_a, info)

call psb_spsm (alpha, t, x, beta, y, desc_a, info, trans, unit, choice, diag, work)

$T, x, y, D, \alpha, \beta$	Subroutine
Long Precision Real	psb_spsm
Long Precision Complex	psb_spsm

Table 12: Data types

On Entry

alpha the scalar α . Scope: global Type: required Specified as: a number of the data type indicated in Table 12. \mathbf{t} the global portion of the sparse matrix T.

Scope: **local** Type: **required** Specified as: a structured data type specified in § 3.

x the local portion of global dense matrix x.
Scope: local
Type: required
Specified as: a rank one or two array containing numbers of type specified in Table 12. The rank of x must be the same of y.

beta the scalar β . Scope: global Type: required Specified as: a number of the data type indicated in Table 12.

 \mathbf{y} the local portion of global dense matrix y.

Scope: local Type: required Specified as: a rank one or two array containing numbers of type specified in Table 12. The rank of y must be the same of x.

desc_a contains data structures for communications. Scope: **local** Type: **required**

Specified as: a structured data of type psb_desc_type.

trans specify with *unitd* the operation to perform.

trans = 'N' the operation is with no transposed matrix

trans = 'T' the operation is with transposed matrix.

trans = 'C' the operation is with conjugate transposed matrix.

Scope: global Type: optional Default: trans = NSpecified as: a character variable.

unitd specify with *trans* the operation to perform.

unitd = 'U' the operation is with no scaling
unitd = 'L' the operation is with left scaling
unitd = 'R' the operation is with right scaling.

Scope: global Type: optional Default: unitd = USpecified as: a character variable.

choice specifies the update of overlap elements to be performed on exit:

psb_none_

psb_sum_
psb_avg_
psb_square_root_

Scope: global Type: optional Default: psb_avg_ Specified as: an integer variable.

diag the diagonal scaling matrix. Scope: local Type: optional Default: diag(1) = 1(noscaling)Specified as: a rank one array containing numbers of the type indicated in Table 12.

 $\mathbf{work}\xspace$ a work array.

Scope: local Type: optional Specified as: a rank one array of the same type of x with the TARGET attribute.

On Return

 \mathbf{y} the local portion of global dense matrix y.

Scope: **local** Type: **required** Specified as: an array of rank one or two containing numbers of type specified in Table 12.

 $\mathbf{info} \ \mathrm{Error} \ \mathrm{code}.$

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

5 Communication routines

The routines in this chapter implement various global communication operators on vectors associated with a discretization mesh. For auxiliary communication routines not tied to a discretization space see 6.

psb_halo—Halo Data Communication

These subroutines gathers the values of the halo elements, and (optionally) scale the result:

 $x \gets \alpha x$

where:

x is a global dense submatrix.

α, x	Subroutine
Long Precision Real	psb_halo
Long Precision Complex	psb_halo

Table 13: Data types

Syntax

call psb_halo $(x, desc_a, info)$

call psb_halo (x, desc_a, info, alpha, work, data)

On Entry

x global dense matrix x.
 Scope: local
 Type: required
 Specified as: a rank one or two array with the TARGET attribute containing numbers of type specified in Table 13.

 ${\bf desc_a}$ contains data structures for communications.

Scope: **local** Type: **required** Specified as: a structured data of type **psb_desc_type**.

alpha the scalar α .

Scope: global Type: optional Default: alpha = 1Specified as: a number of the data type indicated in Table 13.

work the work array.

Scope: local Type: optional Specified as: a rank one array of the same type of x with the POINTER attribute. data index list selector.

Scope: global

Type: optional

Specified as: an integer. Values:psb_comm_halo_, psb_comm_ext_, default: psb_comm_halo_. Chooses the index list on which to base the data exchange.

On Return

 \mathbf{x} global dense result matrix x.

Scope: local

Type: $\mathbf{required}$

Returned as: a rank one or two array containing numbers of type specified in Table 13.

info the local portion of result submatrix y.

Scope: local

Type: required

An integer value that contains an error code.

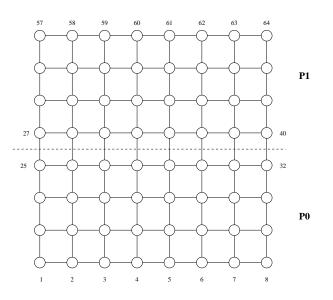


Figure 6: Sample discretization mesh.

Example of use

Consider the discretization mesh depicted in fig. 6, partitioned among two processes as shown by the dashed line; the data distribution is such that each process will own 32 entries in the index space, with a halo made of 8 entries placed at local indices 33 through 40. If process 0 assigns an initial value of 1 to its entries in the x vector, and process 1 assigns a value of 2, then after a call to psb_halo the contents of the local vectors will be the following:

	Proces	ss 0		Process 1	
Ι	GLOB(I)	X(I)	Ι	GLOB(I)	X(I)
1	1	1.0	1	33	2.0
2	2	1.0	2	34	2.0
3	3	1.0	3	35	2.0
4	4	1.0	4	36	2.0
5	5	1.0	5	37	2.0
6	6	1.0	6	38	2.0
7	7	1.0	7	39	2.0
8	8	1.0	8	40	2.0
9	9	1.0	9	41	2.0
10	10	1.0	10	42	2.0
11	11	1.0	11	43	2.0
12	12	1.0	12	44	2.0
13	13	1.0	13	45	2.0
14	14	1.0	14	46	2.0
15	15	1.0	15	47	2.0
16	16	1.0	16	48	2.0
17	17	1.0	17	49	2.0
18	18	1.0	18	50	2.0
19	19	1.0	19	51	2.0
20	20	1.0	20	52	2.0
21	21	1.0	21	53	2.0
22	22	1.0	22	54	2.0
23	23	1.0	23	55	2.0
24	24	1.0	24	56	2.0
25	25	1.0	25	57	2.0
26	26	1.0	26	58	2.0
27	27	1.0	27	59	2.0
28	28	1.0	28	60	2.0
29	29	1.0	29	61	2.0
30	30	1.0	30	62	2.0
31	31	1.0	31	63	2.0
32	32	1.0	32	64	2.0
33	33	2.0	33	25 26	1.0
34	34	2.0	34	26	1.0
35 26	35	2.0	35	27	1.0
$\frac{36}{27}$	36 27	2.0	36 27	28 20	1.0
37	37	2.0	37	29 20	1.0
$\frac{38}{39}$	38 20	2.0	38 20	30 21	1.0
	39 40	2.0	39 40	31 22	1.0
40	40	2.0	40	32	1.0

psb_ovrl —Overlap Update

These subroutines applies an overlap operator to the input vector:

 $x \leftarrow Q x$

where:

 $\boldsymbol{x}\,$ is the global dense submatrix \boldsymbol{x}

Q is the overlap operator; it is the composition of two operators P_a and P^T .

x	Subroutine
Long Precision Real	psb_ovrl
Long Precision Complex	psb_ovrl

Table 14: Data types

Syntax

call psb_ovrl (x, desc_a, info)

call psb_ovrl (x, desc_a, info, update=update_type, work=work)

On Entry

x global dense matrix x.
 Scope: local
 Type: required
 Specified as: a rank one or two array containing numbers of type specified in Table 14.

desc_a contains data structures for communications. Scope: local Type: required Specified as: a structured data of type psb_desc_type.

update Update operator.

update = psb_none_ Do nothing; update = psb_add_ Sum overlap entries, i.e. apply P^T ; update = psb_avg_ Average overlap entries, i.e. apply P_aP^T ;

Scope: global Default: update_type = psb_avg_ Scope: global Specified as: a integer variable. work the work array. Scope: local Type: optional Specified as: a one dimensional array of the same type of x.

On Return

 \mathbf{x} global dense result matrix x. Scope: **local** Type: **required** Specified as: an array of rank one or two containing numbers of type specified in Table 14.

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

Usage notes

- 1. If there is no overlap in the data distribution associated with the descriptor, no operations are performed;
- 2. The operator P^T performs the reduction sum of overlap elements; it is a "prolongation" operator P^T that replicates overlap elements, accounting for the physical replication of data;
- 3. The operator P_a performs a scaling on the overlap elements by the amount of replication; thus, when combined with the reduction operator, it implements the average of replicated elements over all of their instances.

Example of use

Consider the discretization mesh depicted in fig. 7, partitioned among two processes as shown by the dashed lines, with an overlap of 1 extra layer with respect to the partition of fig. 6; the data distribution is such that each process will own 40 entries in the index space, with an overlap of 16 entries placed at local indices 25 through 40; the halo will run from local index 41 through local index 48. If process 0 assigns an initial value of 1 to its entries in the x vector, and process 1 assigns a value of 2, then after a call to psb_ovrl with psb_avg_ and a call to psb_halo_ the contents of the local vectors will be the following (showing a transition among the two subdomains)

	Proce	ss 0		Process 1	
Ι	GLOB(I)	X(I)	Ι	GLOB(I)	X(I)
1	1	1.0	1	33	1.5
2	2	1.0	2	34	1.5
3	3	1.0	3	35	1.5
4	4	1.0	4	36	1.5
5	5	1.0	5	37	1.5
6	6	1.0	6	38	1.5
7	7	1.0	7	39	1.5
8	8	1.0	8	40	1.5
9	9	1.0	9	41	2.0
10	10	1.0	10	42	2.0
11	11	1.0	11	43	2.0
12	12	1.0	12	44	2.0
13	13	1.0	13	45	2.0
14	14	1.0	14	46	2.0
15	15	1.0	15	47	2.0
16	16	1.0	16	48	2.0
17	17	1.0	17	49	2.0
18	18	1.0	18	50	2.0
19	19	1.0	19	51	2.0
20	20	1.0	20	52	2.0
21	21	1.0	21	53	2.0
22	22	1.0	22	54	2.0
23	23	1.0	23	55	2.0
24	24	1.0	24	56	2.0
25	25	1.5	25	57	2.0
26	26	1.5	26	58	2.0
27	27	1.5	27	59	2.0
28	28	1.5	28	60	2.0
29	29	1.5	29	61	2.0
30	30	1.5	30	62	2.0
31	31	1.5	31	63	2.0
32	32	1.5	32	64	2.0
33	33	1.5	33	25	1.5
34	34	1.5	34	26 07	1.5
35	35	1.5	35	27	1.5
36 27	36	1.5	36	28	1.5
37	37	1.5	37	29 20	1.5
38 20	38	1.5	38	30	1.5
39 40	39	1.5	39	31	1.5
40	40	1.5	40	32	1.5
41	41	2.0	41	17	1.0
42 43	42	$2.0 \\ 2.0$	$42 \\ 43$	18 10	1.0
$43 \\ 44$	$43 \\ 44$	$2.0 \\ 2.0$	43 44	19 20	1.0 1.0
$\frac{44}{45}$	$44 \\ 45$	$2.0 \\ 2.0$	$44 \\ 45$	20 21	$1.0 \\ 1.0$
$43 \\ 46$	$43 \\ 46$	$2.0 \\ 2.0$	$43 \\ 46$	$\frac{21}{22}$	$1.0 \\ 1.0$
$\frac{40}{47}$	40 47	$2.0 \\ 2.0$	$40 \\ 47$	22	$1.0 \\ 1.0$
$\frac{47}{48}$	47 48	$2.0 \\ 2.0$	47 48	$\frac{25}{24}$	1.0
40	40	2.0	40	24	1.0

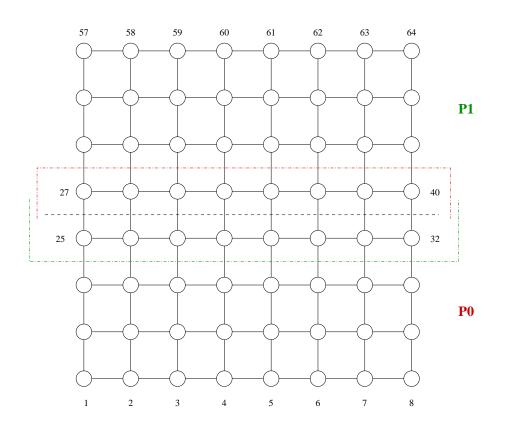


Figure 7: Sample discretization mesh.

psb_gather—Gather Global Dense Matrix

These subroutines collect the portions of global dense matrix distributed over all process into one single array stored on one process.

 $glob_x \leftarrow collect(loc_x_i)$

where:

 $glob_x$ is the global submatrix $glob_x_{iy:iy+m-1,jy:jy+n-1}$

 loc_x_i is the local portion of global dense matrix on process i.

 $collect\,$ is the collect function.

x_i, y	Subroutine
Long Precision Real	psb_gather
Long Precision Complex	psb_gather

Table 15: Data types

Syntax

call psb_gather (glob_x, loc_x, desc_a, info, root, iglobx, jglobx, ilocx, jlocx, k)

Syntax

call psb_gather (glob_x, loc_x, desc_a, info, root, iglobx, ilocx)

On Entry

loc_x the local portion of global dense matrix glob_x. Scope: local Type: required Specified as: a rank one or two array containing numbers of the type indicated in Table 15.
desc_a contains data structures for communications. Scope: local Type: required Specified as: a structured data of type psb_desc_type.
root The process that holds the global copy. If root = -1 all the processes will have a copy of the global vector.

Scope: global Type: optional Specified as: an integer variable $-1 \le ix \le np - 1$, default -1. iglobx Row index to define a submatrix in glob_x into which gather the local pieces. Scope: global

Type: optional

Specified as: an integer variable $1 \le ix \le matrix_data(psb_m_)$.

jglobx Column index to define a submatrix in glob_x into which gather the local pieces. Scope: global Type: optional Specified as: an integer variable.

ilocx Row index to define a submatrix in loc_x that has to be gathered into glob_x. Scope: local Type: optional

Specified as: an integer variable.

jlocx Columns index to define a submatrix in loc_x that has to be gathered into glob_x. Scope: global Type: optional Specified as: an integer variable.

k The number of columns to gather.
 Scope: global
 Type: optional
 Specified as: an integer variable.

On Return

glob_x The array where the local parts must be gathered. Scope: global Type: required Specified as: a rank one or two array.

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

psb_scatter—Scatter Global Dense Matrix

These subroutines scatters the portions of global dense matrix owned by a process to all the processes in the processes grid.

 $loc_x_i \leftarrow scatter(glob_x_i)$

where:

 $glob_x$ is the global submatrix $glob_x_{iy:iy+m-1,jy:jy+n-1}$

 loc_x_i is the local portion of global dense matrix on process i.

scatter is the scatter function.

$\overline{x_i, y}$	Subroutine
Long Precision Real	psb_scatter
Long Precision Complex	$psb_scatter$

Table 16: Data types

Syntax

call psb_scatter (glob_x, loc_x, desc_a, info, root, iglobx, jglobx, ilocx, jlocx, k)

Syntax

call psb_scatter (glob_x, loc_x, desc_a, info, root, iglobx, ilocx)

On Entry

- glob_x The array that must be scattered into local pieces. Scope: global Type: required Specified as: a rank one or two array.
- desc_a contains data structures for communications. Scope: local Type: required Specified as: a structured data of type psb_desc_type.
- **root** The process that holds the global copy. If root = -1 all the processes have a copy of the global vector. Scope: **global** Type: **optional** Specified as: an integer variable $-1 \le ix \le np - 1$, default -1.

iglobx Row index to define a submatrix in glob_x that has to be scattered into local pieces. Scope: global

Type: optional

Specified as: an integer variable $1 \le ix \le matrix_data(psb_m_)$.

jglobx Column index to define a submatrix in glob_x that has to be scattered into local pieces. Scope: global Type: optional Specified as: an integer variable.

ilocx Row index to define a submatrix in loc_x into which scatter the local piece of glob_x. Scope: local Type: optional Specified as: an integer variable.

jlocx Columns index to define a submatrix in loc_x into which scatter the local piece of glob_x. Scope: global Type: optional Specified as: an integer variable.

k The number of columns to scatter. Scope: global Type: optional Specified as: an integer variable.

On Return

loc_x the local portion of global dense matrix glob_x.
Scope: local
Type: required
Specified as: a rank one or two array containing numbers of the type indicated in Table 16.

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

6 Data management routines

psb_cdall—Allocates a communication descriptor

Syntax

call psb_cdall (icontxt, desc_a, info,mg=mg,parts=parts)
call psb_cdall (icontxt, desc_a, info,vg=vg,flag=flag)
call psb_cdall (icontxt, desc_a, info,vl=vl)
call psb_cdall (icontxt, desc_a, info,nl=nl)

This subroutine initializes the communication descriptor associated with an index space. Exactly one of the optional arguments parts, vg, vl or nl must be specified, thereby choosing the specific initialization strategy:

On Entry

icontxt the communication context. Scope:global. Type:required. Specified as: an integer value.

- **vg** Data allocation: each index $i \in \{1...mg\}$ is allocated to process vg(i). Scope:global. Type:optional. Specified as: an integer array.
- **flag** Specifies whether entries in vg are zero- or one-based. Scope:global. Type:optional. Specified as: an integer value 0, 1, default 0.
- mg the (global) number of rows of the problem. Scope:global. Type:optional. Specified as: an integer value. It is required if parts is specified.

parts the subroutine that defines the partitioning scheme. Scope:global. Type:required. Specified as: a subroutine.

 vl Data allocation: the set of global indices belonging to the calling process. Scope:local. Type:optional. Specified as: an integer array. nl Data allocation: in a generalized block-row distribution the number of indices belonging to the current process. Scope:local.
 Type:optional.
 Specified as: an integer value.

On Return

desc_a the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

info Error code. Scope: local Type: required An integer value; 0 means no error has been detected.

Notes

- 1. Exactly one of the optional arguments parts, vg, vl, nl must be specified, thereby choosing the initialization strategy as follows:
 - **parts** In this case we have a subroutine specifying the mapping between global indices and process/local index pairs. If this optional argument is specified, then it is mandatory to specify the argument mg as well. The subroutine must conform to the following interface:

```
interface
   subroutine psb_parts(glob_index,mg,np,pv,nv)
      integer, intent (in) :: glob_index,np,mg
      integer, intent (out) :: nv, pv(*)
   end subroutine psb_parts
end interface
```

The input arguments are:

glob_index The global index to be mapped;

np The number of processes in the mapping;

mg The total number of global rows in the mapping;

The output arguments are:

- nv The number of entries in pv;
- **pv** A vector containing the indices of the processes to which the global index should be assigned; each entry must satisfy $0 \leq pv(i) < np$; if nv > 1 we have an index assigned to multiple processes, i.e. we have an overlap among the subdomains.
- **vg** In this case the association between an index and a process is specified via an integer vector; the size of the index space is equal to the size of **vg**, and each index *i* is assigned to the process vg(i). The vector **vg** must be identical on all calling processes; its entries may have the ranges $(0 \dots np 1)$ or $(1 \dots np)$ according to the value of **flag**.

- **vl** In this case we are specifying the list of indices assigned to the current process; thus, the global problem size mg is given by the sum of the sizes of the individual vectors **vl** specified on the calling processes. The subroutine will check that each entry in the global index space $(1 \dots mg)$ is specified exactly once.
- **nl** In this case we are implying a generalized row-block distribution in which each process I gets assigned a consecutive chunk of $N_I = nl$ global indices.
- 2. On exit from this routine the descriptor is in the build state

psb_cdins—Communication descriptor insert routine

Syntax

call psb_cdins (nz, ia, ja, desc_a, info)

This subroutine examines the edges of the graph associated with the discretization mesh (and isomorphic to the sparsity pattern of a linear system coefficient matrix), storing them as necessary into the communication descriptor.

On Entry

nz the number of points being inserted.
 Scope: local.
 Type: required.
 Specified as: an integer value.

ia the indices of the starting vertex of the edges being inserted.

Scope: **local**. Type: **required**. Specified as: an integer array of length *nz*.

ja the indices of the end vertex of the edges being inserted.
Scope: local.
Type: required.
Specified as: an integer array of length nz.

On Return

desc_a the updated communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

info Error code. Scope: local Type: required An integer value; 0 means no error has been detected.

Notes

- 1. This routine may only be called if the descriptor is in the build state;
- 2. This routine automatically ignores edges that do not insist on the current process, i.e. edges for which neither the starting nor the end vertex belong to the current process.

$psb_cdasb_Communication\ descriptor\ assembly\ routine$

Syntax

call psb_cdasb (desc_a, info)

On Entry

desc_a the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

On Return

info Error code. Scope: local Type: required An integer value; 0 means no error has been detected.

Notes

1. On exit from this routine the descriptor is in the assembled state.

psb_cdcpy—Copies a communication descriptor

Syntax

call psb_cdcpy (desc_out, desc_a, info)

On Entry

desc_a the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

On Return

desc_out the communication descriptor copy. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

info Error code. Scope: local Type: required An integer value; 0 means no error has been detected.

psb_cdfree—Frees a communication descriptor

Syntax

call psb_cdfree (desc_a, info)

On Entry

desc_a the communication descriptor to be freed.
 Scope:local.
 Type:required.
 Specified as: a structured data of type psb_desc_type.

On Return

info Error code.
 Scope: local
 Type: required
 An integer value; 0 means no error has been detected.

psb_cdbldext—Build an extended communication descriptor

Syntax

call psb_cdbldext (a,desc_a,nl,desc_out, info, extype)

This subroutine builds an extended communication descriptor, based on the input descriptor desc_a and on the stencil specified through the input sparse matrix a.

On Entry

a A sparse matrix Scope:local. Type:required. Specified as: a structured data type.

desc_a the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_spmat_type.

nl the number of additional layers desired. Scope:**global**. Type:**required**. Specified as: an integer value $nl \ge 0$.

extype the kind of estension required. Scope:**global**.

> Type:optional . Specified as: an integer value psb_ovt_xhal_, psb_ovt_asov_, default: psb_ovt_xhal_

On Return

desc_out the extended communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

info Error code. Scope: local Type: required An integer value; 0 means no error has been detected.

Notes

- 1. Specifying psb_ovt_xhal_ for the extype argument the user will obtain a descriptor for a domain partition in which the additional layers are fetched as part of an (extended) halo; however the index-to-process mapping is identical to that of the base descriptor;
- 2. Specifying psb_ovt_asov_ for the extype argument the user will obtain a descriptor with an overlapped decomposition: the additional layer is aggregated to the local subdomain (and thus is an overlap), and a new halo extending beyond the last additional layer is formed.

psb_spall—Allocates a sparse matrix

Syntax

call psb_spall (a, desc_a, info, nnz)

On Entry

desc_a the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

nnz An estimate of the number of nonzeroes in the local part of the assembled matrix.
Scope: global.
Type: optional.
Specified as: an integer value.

On Return

a the matrix to be allocated. Scope:local Type:required Specified as: a structured data of type psb_spmat_type.

info Error code.
Scope: local
Type: required
An integer value; 0 means no error has been detected.

Notes

- 1. On exit from this routine the sparse matrix is in the build state.
- 2. The descriptor may be in either the build or assembled state.
- 3. Providing a good estimate for the number of nonzeroes *nnz* in the assembled matrix may substantially improve performance in the matrix build phase, as it will reduce or eliminate the need for (potentially multiple) data reallocations.

psb_spins—Insert a cloud of elements into a sparse matrix

Syntax

call psb_spins (nz, ia, ja, val, a, desc_a, info)

On Entry

- nz the number of elements to be inserted. Scope:local. Type:required. Specified as: an integer scalar.
- ia the row indices of the elements to be inserted. Scope:local. Type:required. Specified as: an integer array of size nz.

ja the column indices of the elements to be inserted.Scope:local.Type:required.Specified as: an integer array of size nz.

val the elements to be inserted. Scope:local. Type:required. Specified as: an array of size nz.

desc_a The communication descriptor.
 Scope: local.
 Type: required.
 Specified as: a variable of type psb_desc_type.

On Return

a the matrix into which elements will be inserted. Scope:local Type:required Specified as: a structured data of type psb_spmat_type.

desc_a The communication descriptor. Scope: local. Type: required. Specified as: a variable of type psb_desc_type. info Error code.
Scope: local
Type: required
An integer value; 0 means no error has been detected.

Notes

- 1. On entry to this routine the descriptor may be in either the build or assembled state.
- 2. On entry to this routine the sparse matrix may be in either the build or update state.
- 3. If the descriptor is in the build state, then the sparse matrix must also be in the build state; the action of the routine is to (implicitly) call psb_cdins to add entries to the sparsity pattern; each sparse matrix entry implicitly defines a graph edge, that is passed to the descriptor routine for the appropriate processing.
- 4. If the descriptor is in the assembled state, then any entries in the sparse matrix that would generate additional communication requirements will be ignored;
- 5. If the matrix is in the update state, any entries in positions that were not present in the original matrix will be ignored.

psb_spasb—Sparse matrix assembly routine

Syntax

call psb_spasb (a, desc_a, info, afmt, upd, dupl)

On Entry

 $\mathbf{desc_a}$ the communication descriptor.

Scope:**local**. Type:**required**. Specified as: a structured data of type **psb_desc_type**.

afmt the storage format for the sparse matrix. Scope: **global**. Type: **optional**. Specified as: an array of characters. Defalt: 'CSR'.

upd Provide for updates to the matrix coefficients. Scope: global. Type: optional. Specified as: integer, possible values: psb_upd_srch_, psb_upd_perm_

dupl How to handle duplicate coefficients. Scope: global. Type: optional. Specified as: integer, possible values: psb_dupl_ovwrt_, psb_dupl_add_, psb_dupl_err_.

On Return

a the matrix to be assembled. Scope:local Type:required Specified as: a structured data of type psb_spmat_type.

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

Notes

- 1. On entry to this routine the descriptor must be in the assembled state, i.e. psb_cdasb must already have been called.
- 2. The sparse matrix may be in either the build or update state;

- 3. Duplicate entries are detected and handled in both build and update state, with the exception of the error action that is only taken in the build state, i.e. on the first assembly;
- 4. If the update choice is psb_upd_perm_, then subsequent calls to psb_spins to update the matrix must be arranged in such a way as to produce exactly the same sequence of coefficient values as encountered at the first assembly;
- 5. On exit from this routine the matrix is in the assembled state, and thus is suitable for the computational routines.

psb_spfree—Frees a sparse matrix

Syntax

call psb_spfree (a, desc_a, info)

On Entry

a the matrix to be freed. Scope:local Type:required Specified as: a structured data of type psb_spmat_type.

desc_a the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

On Return

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

psb_sprn—Reinit sparse matrix structure for psblas routines.

Syntax

call psb_sprn (a, decsc_a, info, clear)

On Entry

- a the matrix to be reinitialized. Scope:local Type:required Specified as: a structured data of type psb_spmat_type.
- desc_a the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.
- **clear** Choose whether to zero out matrix coefficients Scope:**local**. Type:**optional**. Default: true.

On Return

info Error code.
 Scope: local
 Type: required
 An integer value; 0 means no error has been detected.

Notes

1. On exit from this routine the sparse matrix is in the update state.

psb_geall—Allocates a dense matrix

Syntax

call psb_geall $(x, desc_a, info, n)$

On Entry

desc_a The communication descriptor. Scope: local Type: required Specified as: a variable of type psb_desc_type.

n The number of columns of the dense matrix to be allocated.
 Scope: local
 Type: optional
 Specified as: Integer scalar, default 1. It is ignored if x is a rank-1 array.

On Return

 ${\bf x}\,$ The dense matrix to be allocated.

Scope: local Type: required Specified as: a rank one or two array with the ALLOCATABLE attribute, of type real, complex or integer.

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

psb_geins—Dense matrix insertion routine

Syntax

call psb_geins (m, irw, val, x, desc_a, info, dupl)

On Entry

 m Number of rows in *val* to be inserted. Scope:local. Type:required. Specified as: an integer value.

irw Indices of the rows to be inserted. Specifically, row *i* of *val* will be inserted into the local row corresponding to the global row index irw(i). Scope:**local**. Type:**required**. Specified as: an integer array.

val the dense submatrix to be inserted.
Scope:local.
Type:required.
Specified as: a rank 1 or 2 array. Specified as: an integer value.

desc_a the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

dupl How to handle duplicate coefficients. Scope: global. Type: optional. Specified as: integer, possible values: psb_dupl_ovwrt_, psb_dupl_add_.

On Return

x the output dense matrix.
 Scope: local
 Type: required
 Specified as: a rank one or two array with the ALLOCATABLE attribute, of type real, complex or integer.

info Error code.
Scope: local
Type: required
An integer value; 0 means no error has been detected.

Notes

- 1. Dense vectors/matrices do not have an associated state;
- 2. Duplicate entries are either overwritten or added, there is no provision for raising an error condition.

psb_geasb —Assembly a dense matrix

Syntax

call psb_geasb (x, desc_a, info)

On Entry

desc_a The communication descriptor. Scope: local Type: required Specified as: a variable of type psb_desc_type.

On Return

x The dense matrix to be assembled.
 Scope: local
 Type: required
 Specified as: a rank one or two array with the ALLOCATABLE attribute, of type real, complex or integer.

info Error code. Scope: local Type: required An integer value; 0 means no error has been detected.

psb_gefree—Frees a dense matrix

Syntax

call psb_gefree (x, desc_a, info)

On Entry

x The dense matrix to be freed.
 Scope: local
 Type: required
 Specified as: a rank one or two array with the ALLOCATABLE attribute, of type real, complex or integer.

desc_a The communication descriptor. Scope: local Type: required Specified as: a variable of type psb_desc_type.

On Return

info Error code. Scope: local Type: required An integer value; 0 means no error has been detected.

psb_gelp—Applies a left permutation to a dense matrix

Syntax

call psb_gelp (trans, iperm, x, desc_a, info)

On Entry

trans A character that specifies whether to permute A or A^T . Scope: **local** Type: **required** Specified as: a single character with value 'N' for A or 'T' for A^T .

iperm An integer array containing permutation information.Scope: localType: requiredSpecified as: an integer one-dimensional array.

x The dense matrix to be permuted.
 Scope: local
 Type: required
 Specified as: a one or two dimensional array.

desc_a The communication descriptor. Scope: local Type: required Specified as: a variable of type psb_desc_type.

On Return

info Error code.
 Scope: local
 Type: required
 An integer value; 0 means no error has been detected.

psb_glob_to_loc—Global to local indices convertion

Syntax

call psb_glob_to_loc (x, y, desc_a, info, iact, owned)

call psb_glob_to_loc (x, desc_a, info, iact, owned)

On Entry

 x An integer vector of indices to be converted. Scope: local
 Type: required
 Specified as: a rank one integer array.

desc_a the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

iact specifies action to be taken in case of range errors. Scope: globalType: optionalSpecified as: a character variable Ignore, Warning or Abort, default Ignore.

owned Specfies valid range of input Scope: global Type: optionalIf true, then only indices strictly owned by the current process are considered valid, if false then halo indices are also accepted. Default: false.

On Return

x If y is not present, then x is overwritten with the translated integer indices. Scope: **global** Type: **required** Specified as: a rank one integer array.

 \mathbf{y} If y is present, then y is overwritten with the translated integer indices, and x is left unchanged. Scope: **global** Type: **optional** Specified as: a rank one integer array.

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

- 1. If an input index is out of range, then the corresponding output index is set to a negative number;
- 2. The default Ignore means that the negative output is the only action taken on an out-of-range input.

psb_loc_to_glob—Local to global indices conversion

Syntax

call psb_loc_to_glob (x, y, desc_a, info, iact)

call psb_loc_to_glob (x, desc_a, info, iact)

On Entry

 x An integer vector of indices to be converted. Scope: local Type: required Specified as: a rank one integer array.

desc_a the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

iact specifies action to be taken in case of range errors. Scope: global Type: optional Specified as: a character variable Ignore, Warning or Abort, default Ignore.

On Return

x If y is not present, then x is overwritten with the translated integer indices. Scope: **global** Type: **required** Specified as: a rank one integer array.

y If y is not present, then y is overwritten with the translated integer indices, and x is left unchanged. Scope: **global** Type: **optional** Specified as: a rank one integer array.

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

psb_get_boundary—Extract list of boundary elements

Syntax

call psb_get_boundary (bndel, desc, info)

On Entry

desc the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

On Return

bndel The list of boundary elements on the calling process, in local numbering.
Scope: local
Type: required
Specified as: a rank one array with the ALLOCATABLE attribute, of type integer.

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

- 1. If there are no boundary elements (i.e., if the local part of the connectivity graph is self-contained) the output vector is set to the "not allocated" state.
- 2. Otherwise the size of bndel will be exactly equal to the number of boundary elements.

psb_get_overlap—Extract list of overlap elements

Syntax

call psb_get_overlap (ovrel, desc, info)

On Entry

desc the communication descriptor. Scope:local. Type:required. Specified as: a structured data of type psb_desc_type.

On Return

ovrel The list of overlap elements on the calling process, in local numbering.

Scope: **local** Type: **required**

Specified as: a rank one array with the ALLOCATABLE attribute, of type integer.

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

- 1. If there are no overlap elements the output vector is set to the "not allocated" state.
- 2. Otherwise the size of ovrel will be exactly equal to the number of overlap elements.

psb_sp_getrow—Extract row(s) from a sparse matrix

Syntax

call psb_sp_getrow (row, a, nz, ia, ja, val, info, append, nzin, lrw)

On Entry

row The (first) row to be extracted. Scope:**local** Type:**required** Specified as: an integer > 0.

a the matrix from which to get rows.

Scope:**local** Type:**required** Specified as: a structured data of type psb_spmat_type.

append Whether to append or overwrite existing output.

Scope:**local** Type:**optional** Specified as: a logical value default: false (overwrite).

nzin Input size to be appended to.
Scope:local
Type:optional
Specified as: an integer > 0. When append is true, specifies how many entries in the output vectors are already filled.

lrw The last row to be extracted. Scope:local Type:optional Specified as: an integer > 0, default: row.

On Return

 nz the number of elements returned by this call. Scope:local. Type:required. Returned as: an integer scalar.

ia the row indices.
 Scope:local.
 Type:required.
 Specified as: an integer array with the ALLOCATABLE attribute.

ja the column indices of the elements to be inserted.Scope:local.Type:required.Specified as: an integer array with the ALLOCATABLE attribute.

val the elements to be inserted.

Scope:**local**. Type:**required**. Specified as: a real array with the **ALLOCATABLE** attribute.

info Error code.
 Scope: local
 Type: required
 An integer value; 0 means no error has been detected.

- 1. The output nz is always the size of the output generated by the current call; thus, if append=.true., the total output size will be nzin + nz, with the newly extracted rows stored in entries nzin+1:nzin+nz of the array arguments;
- 2. When append=.true. the output arrays are reallocated as necessary;
- 3. The row and column indices are returned in the local numbering scheme; if the global numbering is desired, the user may emply the psb_loc_to_glob routine.

7 Parallel environment routines

psb_init—Initializes PSBLAS parallel environment

Syntax

call psb_init (*icontxt*, np)

This subroutine initializes the PSBLAS parallel environment, defining a virtual parallel machine.

On Entry

np Number of processes in the PSBLAS virtual parallel machine. Scope:**global**.

Type:**optional**.

Specified as: an integer value. Default: use all available processes provided by the underlying parallel environment.

On Return

icontxt the communication context identifying the virtual parallel machine. Scope:global.

Type:**required**. Specified as: an integer variable.

- 1. A call to this routine must precede any other PSBLAS call.
- 2. It is an error to specify a value for np greater than the number of processes available in the underlying parallel execution environment.

psb_info—Return information about PSBLAS parallel environment

Syntax

call psb_info (*icontxt*, *iam*, *np*)

This subroutine returns information about the PSBLAS parallel environment, defining a virtual parallel machine.

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global. Type:required. Specified as: an integer variable.

On Return

- iam Identifier of current process in the PSBLAS virtual parallel machine. Scope:local. Type:required. Specified as: an integer value. $-1 \leq iam \leq np - 1$
- np Number of processes in the PSBLAS virtual parallel machine. Scope:global. Type:required. Specified as: an integer variable.

- 1. For processes in the virtual parallel machine the identifier will satisfy $0 \le iam \le np-1;$
- 2. If the user has requested on psb_init a number of processes less than the total available in the parallel execution environment, the remaining processes will have on return iam = -1; the only call involving icontxt that any such process may execute is to psb_exit .

psb_exit—Exit from PSBLAS parallel environment

Syntax

call psb_exit (icontxt)

call psb_exit (*icontxt,close*)

This subroutine exits from the PSBLAS parallel virtual machine.

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global.

Type:**required**. Specified as: an integer variable.

close Whether to close all data structures related to the virtual parallel machine, besides those associated with icontxt.
Scope:global.
Type:optional.
Specified as: a logical variable, default value: true.

- 1. This routine may be called even if a previous call to psb_info has returned with iam = -1; indeed, it it is the only routine that may be called with argument icontxt in this situation.
- 2. If the user wants to use multiple communication contexts in the same program, this routine may be called multiple times to selectively close the contexts with close=.false., while on the last call it should be called with close=.true. to shutdown in a clean way the entire parallel environment.

$psb_get_mpicomm{--}Get\ the\ MPI\ communicator$

Syntax

call psb_get_mpicomm (*icontxt*, *icomm*)

This subroutine returns the MPI communicator associated with a PSBLAS context

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global. Type:required. Specified as: an integer variable.

On Return

icomm The MPI communicator associated with the PSBLAS virtual parallel machine. Scope:global. Type:required.

psb_get_rank —Get the MPI rank

Syntax

call psb_get_rank (rank, icontxt, id)

This subroutine returns the MPI rank of the PSBLAS process id

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global. Type:required. Specified as: an integer variable.

 $\begin{array}{ll} \mbox{id Identifier of a process in the PSBLAS virtual parallel machine.}\\ Scope: \mbox{local.}\\ Type: \mbox{required.}\\ Specified as: an integer value.} \ 0 \leq id \leq np-1 \end{array}$

On Return

rank The MPI rank associated with the PSBLAS process *id*. Scope:**local**. Type:**required**.

psb_wtime —Wall clock timing

Syntax

time = $psb_wtime()$

This function returns a wall clock timer. The resolution of the timer is dependent on the underlying parallel environment implementation.

On Exit

Function value the elapsed time in seconds.
Returned as: a real(kind(1.d0)) integer variable.

$psb_barrier$ —Sinchronization point parallel environment

Syntax

call psb_barrier (*icontxt*)

This subroutine acts as a synchronization point for the PSBLAS parallel virtual machine. As such, it must be called by all participating processes.

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global. Type:required. Specified as: an integer variable.

psb_abort—Abort a computation

Syntax

call psb_abort (*icontxt*)

This subroutine aborts computation on the parallel virtual machine.

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global. Type:required. Specified as: an integer variable.

psb_bcast—Broadcast data

Syntax

call psb_bcast (icontxt, dat, root)

This subroutine implements a broadcast operation based on the underlying communication library.

On Entry

icontxt the communication context identifying the virtual parallel machine.

Scope:**global**. Type:**required**. Specified as: an integer variable.

dat On the root process, the data to be broadcast.

Scope:global.

Type:required.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array, or a character or logical scalar. Type, rank and size must agree on all processes.

root Root process holding data to be broadcast.

Scope:global. Type:optional. Specified as: an integer value $0 \le root \le np - 1$, default 0

On Return

dat On processes other than root, the data to be broadcast.

Scope:global.

Type:required.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array, or a character or logical scalar. Type, rank and size must agree on all processes.

psb_sum—Global sum

Syntax

call psb_sum (icontxt, dat, root)

This subroutine implements a sum reduction operation based on the underlying communication library.

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global. Type:required. Specified as: an integer variable.

dat The local contribution to the global sum. Scope:global. Type:required. Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

root Process to hold the final sum, or -1 to make it available on all processes. Scope:global. Type:optional. Specified as: an integer value $-1 \le root \le np - 1$, default -1.

On Return

dat On destination process(es), the result of the sum operation. Scope:global. Type:required. Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

Notes

psb_max—Global maximum

Syntax

call psb_max (icontxt, dat, root)

This subroutine implements a maximum value reduction operation based on the underlying communication library.

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global.

Type:**required**. Specified as: an integer variable.

dat The local contribution to the global maximum.
Scope:local.
Type:required.
Specified as: an integer or real variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

root Process to hold the final maximum, or -1 to make it available on all processes. Scope:**global**.

Type:**optional**. Specified as: an integer value $-1 \le root \le np - 1$, default -1.

On Return

dat On destination process(es), the result of the maximum operation.

Scope:**global**. Type:**required**.

Specified as: an integer or real variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

Notes

psb_min—Global minimum

Syntax

call psb_min (icontxt, dat, root)

This subroutine implements a minimum value reduction operation based on the underlying communication library.

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global. Type:required. Specified as: an integer variable.

dat The local contribution to the global minimum.

Scope:local.

 $Type: {\bf required}.$

Specified as: an integer or real variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

root Process to hold the final value, or -1 to make it available on all processes. Scope:global. Type:optional. Specified as: an integer value $-1 \le root \le np - 1$, default -1.

On Return

dat On destination process(es), the result of the minimum operation. Scope:global. Type:required.
Specified as: an integer or real variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

Notes

psb_amx—Global maximum absolute value

Syntax

call psb_amx (icontxt, dat, root)

This subroutine implements a maximum absolute value reduction operation based on the underlying communication library.

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global. Type:required.

Specified as: an integer variable.

dat The local contribution to the global maximum. Scope:local. Type:required.
Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

root Process to hold the final value, or -1 to make it available on all processes. Scope:global. Type:optional. Specified as: an integer value $-1 \le root \le np - 1$, default -1.

On Return

dat On destination process(es), the result of the maximum operation. Scope:global. Type:required.
Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

Notes

psb_amn—Global minimum absolute value

Syntax

call psb_amn (icontxt, dat, root)

This subroutine implements a minimum absolute value reduction operation based on the underlying communication library.

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global. Type:required. Specified as: an integer variable.

dat The local contribution to the global minimum.

Scope:**local**. Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

root Process to hold the final value, or -1 to make it available on all processes. Scope:global. Type:optional. Specified as: an integer value $-1 \le root \le np - 1$, default -1.

On Return

dat On destination process(es), the result of the minimum operation. Scope:global. Type:required.
Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array. Type, rank and size must agree on all processes.

Notes

psb_snd—Send data

Syntax

call psb_snd (*icontxt*, dat, dst, m)

This subroutine sends a packet of data to a destination.

On Entry

icontxt the communication context identifying the virtual parallel machine. Scope:global.

Type:**required**. Specified as: an integer variable.

 ${\bf dat}\,$ The data to be sent.

Scope:local.

Type:**required**.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array, or a character or logical scalar. Type and rank must agree on sender and receiver process; if m is not specified, size must agree as well.

 \mathbf{dst} Destination process.

Scope:global. Type:required. Specified as: an integer value $0 \le dst \le np - 1$.

m Number of rows.

Scope:global.

Type:**Optional**.

Specified as: an integer value $0 \le m \le size(dat, 1)$.

When dat is a rank 2 array, specifies the number of rows to be sent independently of the leading dimension size(dat, 1); must have the same value on sending and receiving processes.

On Return

psb_rcv—Receive data

Syntax

call psb_rcv (icontxt, dat, src, m)

This subroutine receives a packet of data to a destination.

On Entry

icontxt the communication context identifying the virtual parallel machine.

Scope:**global**. Type:**required**. Specified as: an integer variable.

src Source process. Scope:**global**. Type:**required**. Specified as: an integer value $0 \le src \le np - 1$.

 ${\bf m}\,$ Number of rows.

Scope:global.

Type:Optional.

Specified as: an integer value $0 \le m \le size(dat, 1)$.

When dat is a rank 2 array, specifies the number of rows to be sent independently of the leading dimension size(dat, 1); must have the same value on sending and receiving processes.

On Return

dat The data to be received. Scope:local.

Type:required.

Specified as: an integer, real or complex variable, which may be a scalar, or a rank 1 or 2 array, or a character or logical scalar. Type and rank must agree on sender and receiver process; if m is not specified, size must agree as well.

8 Error handling

The PSBLAS library error handling policy has been completely rewritten in version 2.0. The idea behind the design of this new error handling strategy is to keep error messages on a stack allowing the user to trace back up to the point where the first error message has been generated. Every routine in the PSBLAS-2.0 library has, as last non-optional argument, an integer info variable; whenever, inside the routine, en error is detected, this variable is set to a value corresponding to a specific error code. Then this error code is also pushed on the error stack and then either control is returned to the caller routine or the execution is aborted, depending on the users choice. At the time when the execution is aborted, an error message is printed on standard output with a level of verbosity than can be chosen by the user. If the execution is not aborted, then, the caller routine checks the value returned in the info variable and, if not zero, an error condition is raised. This process continues on all the levels of nested calls until the level where the user decides to abort the program execution.

Figure 8 shows the layout of a generic psb_foo routine with respect to the PSBLAS-2.0 error handling policy. It is possible to see how, whenever an error condition is detected, the info variable is set to the corresponding error code which is, then, pushed on top of the stack by means of the psb_errpush. An error condition may be directly detected inside a routine or indirectly checking the error code returned returned by a called routine. Whenever an error is encountered, after it has been pushed on stack, the program execution skips to a point where the error condition is handled; the error condition is handled either by returning control to the caller routine or by calling the psb_error routine which prints the content of the error stack and aborts the program execution.

Figure 9 reports a sample error message generated by the PSBLAS-2.0 library. This error has been generated by the fact that the user has chosen the invalid "FOO" storage format to represent the sparse matrix. From this error message it is possible to see that the error has been detected inside the psb_cest subroutine called by psb_spasb ... by process 0 (i.e. the root process).

```
subroutine psb_foo(some args, info)
  if(error detected) then
     info=errcode1
      call psb_errpush('psb_foo', errcode1)
      goto 9999
  end if
   . . .
  call psb_bar(some args, info)
  if(info .ne. zero) then
      info=errcode2
      call psb_errpush('psb_foo', errcode2)
      goto 9999
   end if
9999 continue
  if (err_act .eq. act_abort) then
    call psb_error(icontxt)
    return
  else
    return
  end if
end subroutine psb_foo
```

Figure 8: The layout of a generic psb_foo routine with respect to PSBLAS-2.0 error handling policy.

```
Process: 0. PSBLAS Error (4010) in subroutine: df_sample
Error from call to subroutine mat dist
Process: 0. PSBLAS Error (4010) in subroutine: mat_distv
Error from call to subroutine psb_spasb
Process: 0. PSBLAS Error (4010) in subroutine: psb_spasb
Error from call to subroutine psb_cest
Process: 0. PSBLAS Error (136) in subroutine: psb_cest
Format FOO is unknown
Aborting...
```

Figure 9: A sample PSBLAS-2.0 error message. Process 0 detected an error condition inside the psb_cest subroutine

$psb_errpush$ —Pushes an error code onto the error stack

Syntax

call psb_errpush (err_c, r_name, i_err, a_err)

On Entry

err_c the error code Scope: local Type: required Specified as: an integer.

r_name the soutine where the error has been caught.
 Scope: local
 Type: required
 Specified as: a string.

i_err addional info for error code Scope: **local** Type: **optional** Specified as: an integer array

a_err addional info for error code Scope: **local** Type: **optional** Specified as: a string.

psb_error—Prints the error stack content and aborts execution

Syntax

call psb_error (*icontxt*)

On Entry

icontxt the communication context. Scope: global Type: optional Specified as: an integer.

$psb_set_errverbosity\begin{subarray}{c} Sets the verbosity of error messages. \end{subarray}$

Syntax

call psb_set_errverbosity (v)

On Entry

v the verbosity level Scope: **global** Type: **required** Specified as: an integer.

$psb_set_erraction$ —Set the type of action to be taken upon error condition.

Syntax

call psb_set_erraction (*err_act*)

On Entry

err_act the type of action. Scope: global Type: required Specified as: an integer.

$psb_errcomm$ —Error communication routine

Syntax

call psb_errcomm (*icontxt*, *err*)

On Entry

icontxt the communication context. Scope: global Type: required Specified as: an integer.

err the error code to be communicated Scope: global Type: required Specified as: an integer.

9 Utilities

We have some utilities available for input and output of sparsematrices; the interfaces to these routines are available in the module psb_util_mod.

hb_read—Read a sparse matrix from a file

Syntax

call hb_read (a, iret, iunit, filename, b, mtitle)

On Entry

filename The name of the file to be read.

Type:**optional**.

Specified as: a character variable containing a valid file name, or -, in which case the default input unit 5 (i.e. standard input in Unix jargon) is used. Default: -.

iunit The Fortran file unit number.

Type:**optional**.

Specified as: an integer value. Only meaningful if filename is not -.

On Return

a the sparse matrix read from file.

Type:required.

Specified as: a structured data of type psb_spmat_type.

b Rigth hand side.

Type: Optional

An array of type real or complex, rank 1 and having the ALLOCATABLE attribute; will be allocated and filled in if the input file contains a right hand side.

mtitle Matrix title.

Type: Optional

A charachter variable of length 72 holding a copy of the matrix title as specified by the Harwell-Boeing format and contained in the input file.

iret Error code.

 $Type: \ \mathbf{required}$

An integer value; 0 means no error has been detected.

hb_write—Write a sparse matrix to a file

Syntax

call hb_write (a, iret, iunit, filename, key, rhs, mtitle)

On Entry

a the sparse matrix to be written.

Type:**required**.

Specified as: a structured data of type psb_spmat_type.

 ${\bf b}\,$ Rigth hand side.

Type: Optional

An array of type real or complex, rank 1 and having the ALLOCATABLE attribute; will be allocated and filled in if the input file contains a right hand side.

filename The name of the file to be written to.

Type:**optional**.

Specified as: a character variable containing a valid file name, or -, in which case the default output unit 6 (i.e. standard output in Unix jargon) is used. Default: -.

iunit The Fortran file unit number.

Type:**optional**.

Specified as: an integer value. Only meaningful if filename is not -.

- \mathbf{key} Matrix key.
 - Type: **Optional**

A charachter variable of length 8 holding the matrix key as specified by the Harwell-Boeing format and to be written to file.

mtitle Matrix title.

Type: Optional

A charachter variable of length 72 holding the matrix title as specified by the Harwell-Boeing format and to be written to file.

On Return

iret Error code.

Type: required

An integer value; 0 means no error has been detected.

mm_mat_read—Read a sparse matrix from a file

Syntax

call mm_mat_read (a, iret, iunit, filename)

On Entry

filename The name of the file to be read.

Type:**optional**.

Specified as: a character variable containing a valid file name, or -, in which case the default input unit 5 (i.e. standard input in Unix jargon) is used. Default: -.

iunit The Fortran file unit number.

Type:optional.

Specified as: an integer value. Only meaningful if filename is not -.

On Return

a the sparse matrix read from file. Type:**required**. Specified as: a structured data of type **psb_spmat_type**.

iret Error code.

Type: **required** An integer value; 0 means no error has been detected.

mm_mat_write—Write a sparse matrix to a file

Syntax

call mm_mat_write (a, mtitle, iret, iunit, filename)

On Entry

a the sparse matrix to be written. Type:required. Specified as: a structured data of type psb_spmat_type.

mtitle Matrix title.

Type: **required** A charachter variable holding a descriptive title for the matrix to be written to file.

filename The name of the file to be written to.

Type:**optional**.

Specified as: a character variable containing a valid file name, or -, in which case the default output unit 6 (i.e. standard output in Unix jargon) is used. Default: -.

iunit The Fortran file unit number.

Type:optional.

Specified as: an integer value. Only meaningful if filename is not -.

On Return

iret Error code.

Type: required

An integer value; 0 means no error has been detected.

10 Preconditioner routines

The base PSBLAS library contains the implementation of two simple preconditioning techniques:

- Diagonal Scaling
- Block Jacobi with ILU(0) factorization

The supporting data type and subroutine interfaces are defined in the module psb_prec_mod.

psb_precset—Sets the preconditioner type

Syntax

call psb_precset (prec, ptype, info)

On Entry

prec Scope: **local** Type: **required** Specified as: a pronditioner data structure **psb_prec_type**.

ptype the type of preconditioner. Scope: **global** Type: **required** Specified as: a character string, see usage notes.

On Exit

info Scope: global Type: required Error code: if no error, 0 is returned.

Usage Notes

Legal inputs to this subroutine are interpreted depending on the ptype string as follows²:

NONE No preconditioning, i.e. the preconditioner is just a copy operator.

- **DIAG** Diagonal scaling; each entry of the input vector is multiplied by the reciprocal of the sum of the absolute values of the coefficients in the corresponding row of matrix A;
- **BJAC** Precondition by a factorization of the block-diagonal of matrix A, where block boundaries are determined by the data allocation boundaries for each process; requires no communication. Only the incomplete factorization ILU(0) is currently implemented.

²The string is case-insensitive

psb_precbld—Builds a preconditioner

Syntax

call psb_precbld (a, desc_a, prec, info, upd)

On Entry

- a the system sparse matrix. Scope: local Type: required Specified as: a sparse matrix data structure psb_spmat_type.
- desc_a the problem communication descriptor. Scope: local Type: required Specified as: a communication descriptor data structure psb_desc_type.

upd Scope: global Type: optional Specified as: a character.

On Return

prec the preconditioner. Scope: local Type: required Specified as: a preconditioner data structure psb_prec_type

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

psb_precaply—Preconditioner application routine

Syntax

call psb_precaply (prec,x,y,desc_a,info,trans,work)

call psb_precaply (prec,x,desc_a,info,trans)

On Entry

- prec the preconditioner. Scope: local Type: required Specified as: a preconditioner data structure psb_prec_type.
- **x** the source vector. Scope: **local** Type: **require** Specified as: a double precision array.
- desc_a the problem communication descriptor. Scope: local Type: required Specified as: a communication data structure psb_desc_type.

trans Scope:

Type: **optional** Specified as: a character.

work an optional work space Scope: local
 Type: optional
 Specified as: a double precision array.

On Return

y the destination vector. Scope: **local** Type: **required** Specified as: a double precision array.

info Error code.

Scope: **local** Type: **required** An integer value; 0 means no error has been detected.

$psb_prec_descr_Prints\ a\ description\ of\ current \\ preconditioner$

Syntax

call psb_prec_descr (prec)

On Entry

prec the preconditioner. Scope: local Type: required Specified as: a preconditioner data structure psb_prec_type.

11 Iterative Methods

In this chapter we provide routines for preconditioners and iterative methods. The interfaces for Krylov subspace methods are available in the module psb_krylov_mod.

psb_krylov —Krylov Methods Driver Routine

This subroutine is a driver that provides a general interface for all the Krylov-Subspace family methods implemented in PSBLAS-2.0.

The stopping criterion is the normwise backward error, in the infinity norm, i.e. the iteration is stopped when

$$err = \frac{\|r_i\|}{(\|A\| \|x_i\| + \|b\|)} < eps$$

or the 2-norm residual reduction

$$err = \frac{\|r_i\|}{\|b\|_2} < eps$$

according to the value passed through the istop argument (see later). In the above formulae, x_i is the tentative solution and $r_i = b - Ax_i$ the corresponding residual at the *i*-th iteration.

Syntax

call psb_krylov (method, a, prec, b, x, eps, desc_a, info, itmax, iter, err, itrace, istop)

On Entry

method a string that defines the iterative method to be used. Valid values in PSBLAS-2.0 are:

CG : the Conjugate gradient method;

CGS : the Conjugate Gradient Stabilized method;

BICG : the Bi-Conjugate Gradient method;

- **BICGSTAB** : the Bi-Conjugate Gradient Stabilized method;
- **BICGSTABL** : the Bi-Conjugate Gradient Stabilized method with restarting;

RGMRES : the Generalized Minimal Residual method with restarting.

 \mathbf{a} the local portion of global sparse matrix A.

Scope: local Type: required Specified as: a structured data of type psb_spmat_type.

prec The data structure containing the preconditioner.

Scope: **local** Type: **required** Specified as: a structured data of type **psb_prec_type**.

b The RHS vector.
 Scope: local
 Type: required
 Specified as: a rank one array.

 \mathbf{x} The initial guess. Scope: local Type: required Specified as: a rank one array. eps The stopping tolerance. Scope: global Type: required Specified as: a real number. desc_a contains data structures for communications. Scope: local Type: required Specified as: a structured data of type psb_desc_type. itmax The maximum number of iterations to perform. Scope: global Type: optional Default: itmax = 1000. Specified as: an integer variable $itmax \ge 1$. itrace If > 0 print out an informational message about convergence every *itrace* iterations. Scope: global Type: optional istop An integer specifying the stopping criterion. Scope: global Type: optional. Values: 1: use the normwise backward error, 2: use the scaled 2-norm of the residual. Default: 1. **On Return** ${\bf x}\,$ The computed solution. Scope: local Type: required Specified as: a rank one array. iter The number of iterations performed. Scope: global Type: optional Returned as: an integer variable. err The convergence estimate on exit. Scope: global Type: optional Returned as: a real number. info Error code. Scope: local

Type: **required** An integer value; 0 means no error has been detected.

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