

es. 2.2020 (Gennaio)

$$(i) L_{pal} = \{w \in \{0,1\}^* \mid w = w^R\}$$

$L_{pal}$  regolare  $\Rightarrow 0^n 1^n 0^n = xyz \mid |xy| \leq n, y \neq \epsilon, xy^i z \in L_{pal}$

$\forall i \in \mathbb{N} \Rightarrow xz \in L_{pal} \nleftrightarrow L_{pal}$  non è regolare.

$$P \rightarrow \epsilon \mid 0 \mid 1 \mid 0P0 \mid 1P1$$

$$L_{pal} = L(G):$$

$$\cdot L_{pal} \subset L(G)$$

$$\cdot |w| \leq 1, \text{ base.}$$

$$\cdot |w| = n. \quad w = 0^n: P \Rightarrow d \Rightarrow 0d0 \Rightarrow w.$$

$$w = 1^n: P \Rightarrow d \Rightarrow 1d1 \Rightarrow w.$$

$$\cdot L(G) \subset L_{pal}$$

$$\cdot \epsilon, 0, 1 \in L_{pal}$$

$$\cdot P \Rightarrow 0P0 \mid 1P1. \quad (0P0)^R = 0P^R0 = 0P0.$$

$$(1P1)^R = 1P^R1 = 1P1 \Rightarrow w \in L_{pal}.$$

(ii) Per il PL  $0^n 1^n 2^n$ .

es. 2.2020 (Febbraio)

(i)  $L_1 = \{b^2 a^n b^m a^3 \mid m, n \geq 0\}$   $bba^*b^*aaa$  reg  $\rightarrow$  CFL

(ii)  $L_2 = \{a^n w \mid w \in \{a, b\}^+ \text{ t.c. } |w| = n, n \geq 0\}$

Per il PL,  $a^n b^n$ ; non è regolare.

$P \Rightarrow \varepsilon \mid aPa \mid aPb$ . E' CFL.

(iii)  $L_3 = \{a^i b^j c^k \mid i, j \geq 0, k = \max(i, j)\}$ .

$$a^n b^n c^n = stuvx$$

$$tuv = a^n \Rightarrow st^2 uv^2 x \notin L_3$$

$$tuv = b^n \Rightarrow st^2 uv^2 x \notin L_3$$

$$tuv = c^n \Rightarrow st^2 uv^2 x \notin L_3$$

$$tuv \ni a, b \Rightarrow sux \notin L_3$$

$$tuv \ni b, c \Rightarrow sux \notin L_3$$

$\Rightarrow \neg \text{CFL} (\Rightarrow \neg \text{reg})$

es. 5.2020 (Febbraio)

$$\max(L) = \{ w \in L \mid \nexists x \neq \varepsilon \text{ t.c. } wx \in L \}$$

(i)  $L$  regolare  $\Rightarrow$   $\max(L)$  regolare.

Copiare il DFA di  $L$ . Normalizzare gli stati da cui sono raggiungibili; altri: stati finali.

$$\text{(ii)} \quad L = \{ a^i b^j c^k \mid k \leq i \vee k \leq j \}$$

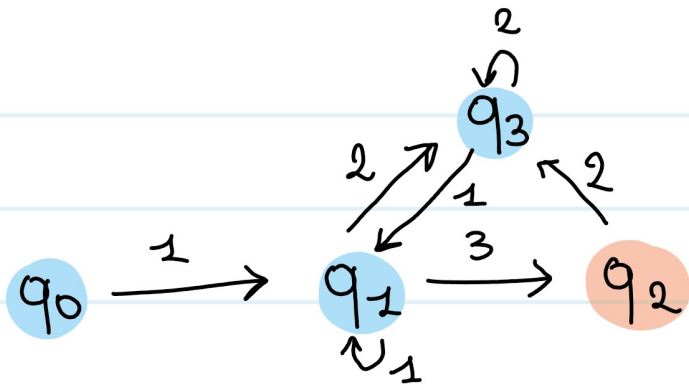
$$P \rightarrow aBc \mid aPc \mid aP$$

$$B \rightarrow b \mid bB$$

$$\text{Max}(L) = \{ a^i b^j c^{\max(i,j)} \} \text{ non CFL.}$$

es. 1.2019 (Gennaio)

$$\Sigma = \{1, 2, 3\}$$



$$1(1+22^*1)^*3(22^*1(1+22^*1)^*3)^*$$

$$P = \{ q_0 \rightarrow 1q_1, q_1 \rightarrow 1q_1 \mid 2q_3 \mid 3q_2, \\ q_2 \rightarrow 2q_3 \mid \epsilon, q_3 \rightarrow 2q_3 \mid 1q_1 \}$$

$$G = (\{q_0, \dots, q_3\}, \{1, 2, 3\}, P, q_0)$$

es. 2.2019 (Gennaio)

$$(i) L_1 = \{ 0^i 1^j 2^j 3^i \mid i \geq 1, j \geq 1 \}$$

Per il PL,  $L_1$  non reg.

$$\left. \begin{array}{l} E \rightarrow 0I3 \\ I \rightarrow 12 \mid 1I2 \end{array} \right\} \text{libero}$$

$$(ii) L_2 = \{0^i 1^j 2^j 3^j \mid i \geq 1, j \geq 1\}$$

Per il PL con  $i=1$ ,  $L_2$  non libero.

es. 2.2019 (Giugno)

$$(i) L_1 = \{0^n 1^{2^n} \mid n \geq 1\}$$

Per PL non è reg. con  $0^n 1^{2^n}$ .

$$E \rightarrow 011 \mid 0E11 \Rightarrow CFL$$

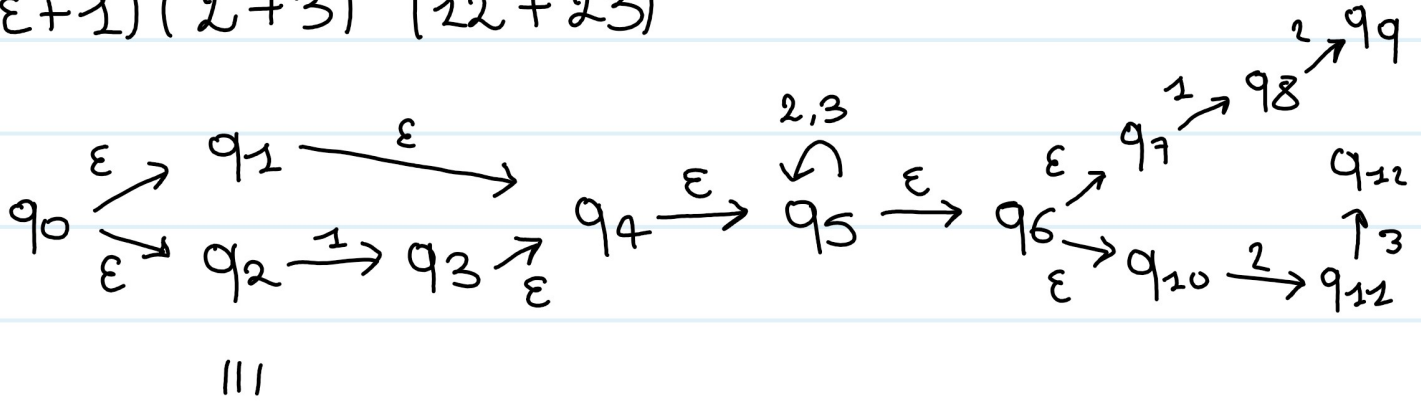
$$(ii) L_2 = \{ww \mid w \in \{a, b\}^*\}$$

$a b^n a b^n$  per il PL  $\Rightarrow \neg CFL \Rightarrow \neg reg.$

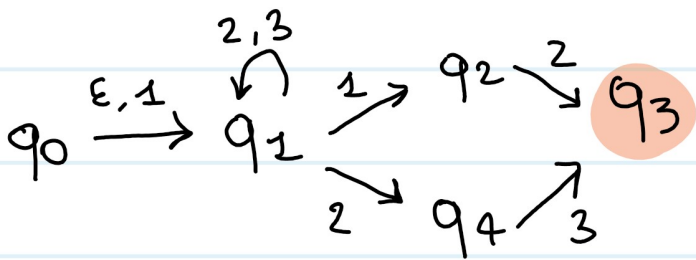
ES. 1. 2018

$$\Sigma = \{1, 2, 3\}$$

$$(\varepsilon+1)(2+3)^*(12+23)$$



|||



$$\{q_0, q_1\} \quad \{q_1\} \quad \{q_2\} \quad \{q_3\} \quad \{q_4\}$$

	1	2	3		1	2	3
$\{q_0, q_1\}$	$\{q_1, q_2\}$	$\{q_1, q_4\}$	$\{q_1\}$	$\{q_3\}$	$\emptyset$	$\emptyset$	$\emptyset$
$\{q_1\}$	$\{q_2\}$	$\{q_1, q_4\}$	$\{q_1\}$	$\{q_4\}$	$\emptyset$	$\emptyset$	$\{q_3\}$
$\{q_2\}$	$\emptyset$	$\{q_3\}$	$\emptyset$				

$$\{q_1, q_2\} \quad \{q_2\} \quad \{q_2, q_3, q_4\} \quad \{q_2\}$$

$$\{q_1, q_4\} \quad \{q_2\} \quad \{q_2, q_4\} \quad \{q_1, q_3\}$$

...

es. 1. 2018 (Giugno)

$$\{0, 1, 2, 3\} \quad \{4\}$$

$$a: \{q_0, q_1, q_3\} \quad \{q_2, q_5\} \quad \{4\}$$

$$b: \{0, 1, 2, 3, 5\} \quad \{4\}$$

$$\{0, 1, 3\} \quad \{2, 5\} \quad \{4\}$$

$$a: \{0, 1, 3\} \quad \{2, 5\} \quad \{4\}$$

$$b: \{0, 3\} \quad \{1\} \quad \{2, 5\} \quad \{4\}$$

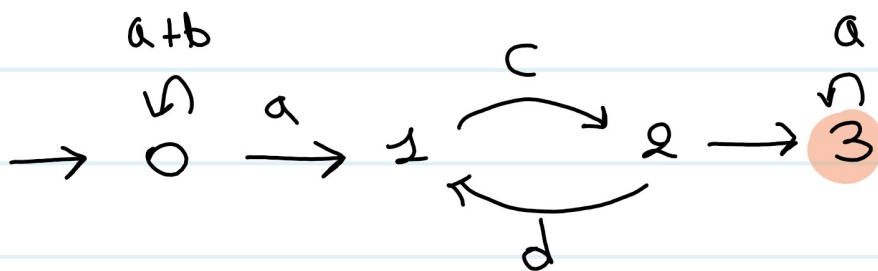
$$\{0, 3\} \quad \{1\} \quad \{2, 5\} \quad \{4\} \checkmark$$

$$a: \quad " \quad " \quad " \quad "$$

b: " " " "

$$\left. \begin{array}{l} E \rightarrow aIc \mid aEc \\ I \rightarrow \varepsilon \mid bIc \end{array} \right\} \begin{array}{l} a^n b^m c^m c^n \\ a^n b^m c^{n+m}, n > 0, m \geq 0 \end{array}$$

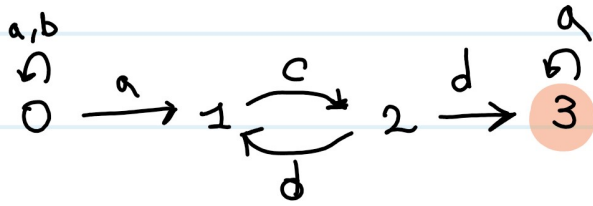
es. 1.2018 (Luglio)



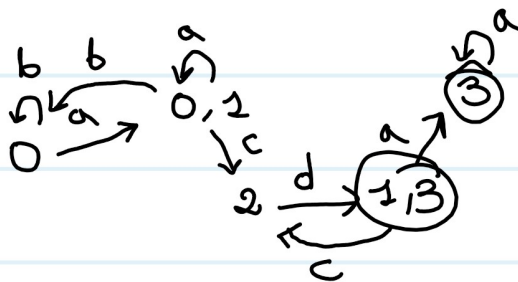
$$(a+b)^* a (cd)^* c a^*$$



ES. 1. 2014 (Gennaio)



$$(a+b)^* a (cd)^* c d a^*$$



	a	b	c	d
0	0,1	0	/	/
1	/	/	2	/
2	/	/	/	1,3
3	3	/	/	/
0,1	0,1	0	2	/
1,3	3	/	2	/

$$\{q_0 \rightarrow bq_0 \mid aq_1, q_1 \rightarrow bq_0 \mid aq_1, q_1 \rightarrow dq_3, q_3 \rightarrow cq_1 \mid aq_4 \mid \epsilon, q_4 \rightarrow aq_4 \mid \epsilon\}$$

ES. 1. 2013 (Gennaio)

$$\Sigma = \{a, b, c\}$$

$$a^{n^+} (a^k b^+) b^{n^+} \equiv a^m b^n, n \geq 2, m \geq 1$$

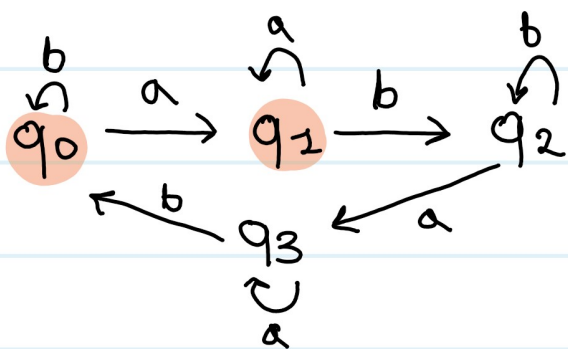
$$S \rightarrow aSb \mid aAb \quad a a^* b^+ b$$

$$A \rightarrow aA \mid B \quad a^* b^+$$

$$B \rightarrow bB \mid b \quad b^+$$

es. 1. 2013 (Luglio)

(i)  $L = \{w \in \{a, b\} \mid \text{occ. di } ab \text{ pari}\}$

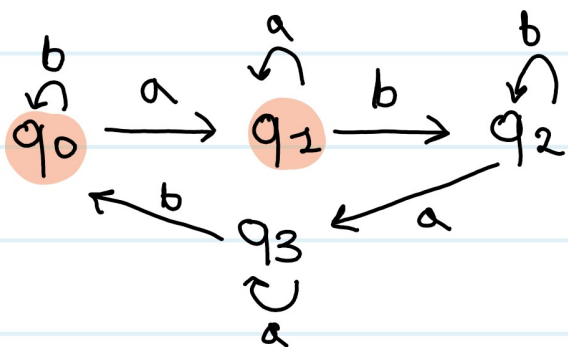


$\{q_0, q_1\} \quad \{q_2, q_3\}$

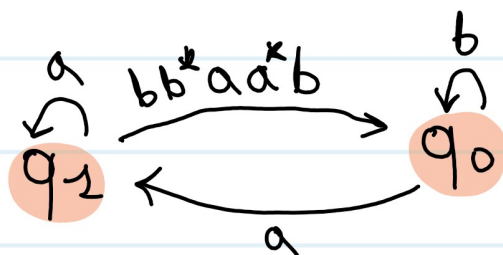
a:  $\{q_0, q_1\} \quad \{q_2, q_3\}$

b:  $\{q_0\} \quad \{q_1\} \quad \{q_2\} \quad \{q_3\}$

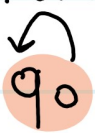
$\{q_0\} \quad \{q_1\} \quad \{q_2\} \quad \{q_3\} \quad \checkmark$



I.



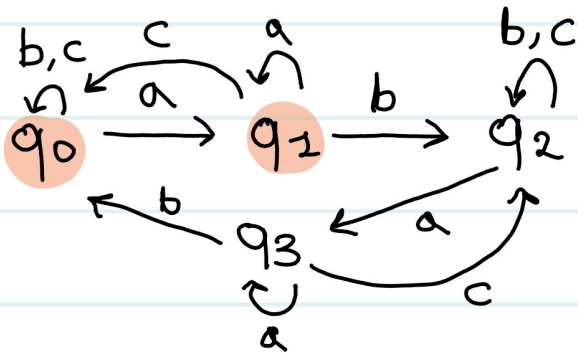
II.

$$b + aa^*(bb^*aa^*b)$$


$$(b + aa^*(bb^*aa^*b))^* (\epsilon + aa^*)$$

$$\{E \rightarrow 1 \mid 1E0 \mid 0E1 \mid 10E \mid 01E \mid E10 \mid E01 \mid 1E \mid E1\}$$

~~11110001~~



$$\{q_0, q_1\} \quad \{q_2, q_3\}$$

001111100

101111100100

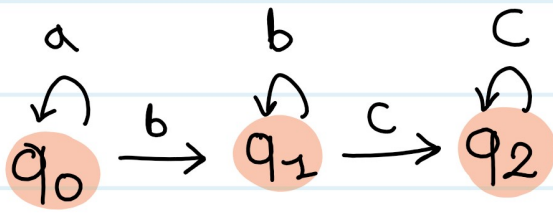
$\{ E \rightarrow A1A \mid EE \mid AEA, \\ A \rightarrow \epsilon \mid 0A1 \mid 1A0 \mid AA \}$

001111100

$\begin{array}{cc} \underbrace{011110}_{A11A} & \underbrace{011110}_{A1A} \\ \underbrace{\quad \quad \quad}_{E} & \underbrace{\quad \quad \quad}_{E} \\ \underbrace{\quad \quad \quad}_{E} & \end{array}$

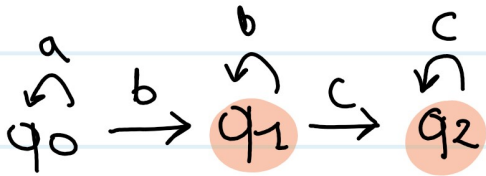
$\underbrace{1001100}_{A} \underbrace{111}_{E}$

$\begin{array}{cc} 0111 \\ \boxed{\quad} \boxed{\quad} \\ T \quad S \end{array}$



$$a^n b^{n+1} c^{n+2}$$

$$\begin{cases} E \rightarrow a I b \mid a E b \\ I \rightarrow \varepsilon \mid b I c \end{cases}$$



es. 2. 2018 (Febbraio)

$$\begin{aligned} 3n &\equiv 0 \pmod{2} \Rightarrow \\ n &\equiv 0 \pmod{2} \end{aligned}$$

$$L = \{ c^{2n} d^{6n} \mid n > 0 \}$$

$$P \rightarrow c^2 d^6 \mid c^2 P d^6$$

$$\begin{cases} E \rightarrow 1T \mid 1E \mid TE \\ T \rightarrow \varepsilon \mid 0T1 \mid 1T0 \mid TT \end{cases}$$