Parte principale

Supponiamo che $f \sim ax^b$, $g \sim cx^d$; allora la pp di $f+g$:

	$x \to 0$	$x \to +\infty$
$b>d$	cx^d	ax^b
$b<d$	ax^b	cx^d
$b=d$	$\underbrace{(a+c)}_{\neq 0} x^b$	$\underbrace{(a+c)}_{\neq 0} x^b$
$a=-c$	indicibile	indicibile

$$x - \frac{x^2}{2} + \dots$$

es.

$(x \to 0)$

- $\sin(x^4) \sim x^4$
- $\log(1+x^2)\sin(x) \sim x^3$
- $\sin(x^2) - \log(1+x^3) \sim x^2 - x^3 \sim x^2$
- $\sin(x^2) - \log(1+x^2) \sim$

$$\sim \cancel{x^2} - \frac{x^6}{3!} + o(x^{10}) - \cancel{x^2} + \frac{x^4}{2} + o(x^6) \sim$$

$$\sim \frac{1}{2}x^4$$

- $\sin(x+x^3) - x \sim (x+x^3) - \frac{(x+x^3)^3}{3!} \sim$

$$\sim \left(1 - \frac{1}{3!}\right)x^3 \sim \frac{5}{6}x^3$$

es.

$(x \to +\infty)$

- $\sqrt[3]{1+x} \sim x^{\frac{1}{3}}$
- $\sqrt[3]{x+1} + \sqrt[3]{x-1} \sim 2x^{1/3}$
- $\sqrt[3]{x+1} - \sqrt[3]{x-1} \sim (A)$

$- (x+1)^{1/3} = x^{\frac{1}{3}}\left(1+\frac{1}{x}\right)^{1/3} \qquad t = \frac{1}{x} \to 0$

$(1+t)^{\frac{1}{3}} \sim 1 + \frac{1}{3}t + O(t^2) \Longrightarrow$

$\Longrightarrow (1+\frac{1}{x})^{\frac{1}{3}} \sim 1 + \frac{1}{3x} + O\left(\frac{1}{x^2}\right) \Longrightarrow$

$\Longrightarrow x^{\frac{1}{3}}\left(1 + \frac{1}{3x} + O\left(\frac{1}{x^2}\right)\right) =$

$= x^{\frac{2}{3}} + \frac{1}{3}x^{-2/3} + x^{2/3}\, O\left(x^{-2}\right) =$

$= x^{1/3} + \frac{1}{3}x^{-2/3} + O\left(x^{1/3}\right)$

$- (x-1)^{1/3} = x^{1/3}\left(1 - \frac{1}{x}\right)^{1/3} \sim$

$\sim x^{2/3} - \frac{1}{3}x^{-2/3} + O\left(x^{1/3}\right)$

$(A) \sim \frac{2}{3}x^{-2/3}$

es

$$f(x) = \frac{\sqrt{1-\cos(2x)}}{e^{x^2}} \qquad a \in \mathbb{R} \quad (x \to 0^+)$$

(i) pp di: $f(x) + ax$ $\qquad \underline{1-\cos(x)} \sim \frac{1}{2}x^2$

$$e^{x^2} \sim 1 \longrightarrow \frac{\sqrt{1-\cos(2x)}}{e^{x^2}} \sim \sqrt{1-\cos(2x)} \sim$$

$$\sim \sqrt{\tfrac{1}{2}(2x)^2} = \sqrt{2x^2} = \sqrt{2}\,x$$

- se $a = -\sqrt{2}$:

$$\sqrt{1-\cos(2x)} \sim \sqrt{2x^2 - \frac{1}{4!}(2x)^4 + O(x^6)} \sim$$

$$\sim \sqrt{2x^2 - \frac{2}{3}x^4 + O(x^6)}$$

$$e^{x^2} \sim 1 + x^2 + O(x^4)$$

$$\sqrt{1-\cos(2x)}\, e^{-x^2} \sim$$

$$\sim \sqrt{2x^2 - \frac{2}{3}x^4 + O(x^6)}\,\left(1 + x^2 + O(x^4)\right)^{-1} \sim$$

$$\sim \sqrt{2x^2 - \frac{2}{3}x^4 + O(x^6)}\,\left(1 - x^2 + O(x^4)\right) \sim$$

$$\sim \sqrt{2}x\left(1 - \frac{1}{3}x^2 + O(x^4)\right)^{1/2}\left(1 - x^2 + O(x^4)\right) \sim$$

$$\sim \sqrt{2}x\left(1 - \frac{1}{6}x^2 + O(x^4)\right)\left(1 - x^2 + O(x^4)\right) \sim$$

$$\sim \sqrt{2}x - \frac{\sqrt{2}}{6}x^3 - \sqrt{2}x^3 + O(x^5) \sim$$

$$\sim \sqrt{2}x - \frac{7}{6}\sqrt{2}x^3 + O(x^5)$$

$$\sqrt{2}\left(x - \frac{x^3}{6} + O(x^5)\right)\left(1 - x^2 + O(x^4)\right) =$$

$$= \sqrt{2}x + \sqrt{2}\left(-\frac{x^3}{6} - x^3\right) + O(x^5)$$

es. $a \in \mathbb{R}$ $\quad f(x) = (x+3a)^a + (x-1)^a - 2x^a$

$(x \to +\infty)$ (i) pp di $f(x)$ $[a \neq 0, \frac{1}{3}]$

(ii) $a = \frac{1}{3}$

(i) $$(x+3a)^a = x^a\left(1+\frac{3a}{x}\right)^a =$$
$$= x^a\left(1+\frac{3a^2}{x}+O\left(\frac{1}{x^2}\right)\right) =$$
$$= x^a + 3a^2x^{a-1} + O(x^{a-2})$$

$$(x-1)^a = x^a\left(1-\frac{1}{x}\right)^a = x^a\left(1-\frac{a}{x}+O\left(\frac{1}{x^2}\right)\right) =$$
$$= x^a - ax^{a-1} + O(x^{a-2})$$

Quindi è $(3a^2-a)x^{a-1}$ (infatti $3a^2-a \neq 0$)

(ii) $$\left(1+\frac{3a}{x}\right)^a = 1+\frac{3a^2}{x}+\frac{a(a-1)}{2}\cdot\left(\frac{3a}{x}\right)^2+O\left(\frac{1}{x^3}\right) =$$
$$= 1+\frac{3a^2}{x}+\frac{9}{2}\,\frac{a^3(a-1)}{x^2}+O\left(\frac{1}{x^3}\right) \Rightarrow$$

$$\Rightarrow x^a + 3a^2x^{a-1} + \frac{9}{2}a^3(a-1)x^{a-2} + O\left(\frac{1}{x^3}\right)$$

$$\left(1-\frac{1}{x}\right)^a = 1-\frac{a}{x}+\frac{a(a-1)}{2}\,\frac{1}{x^2}+O\left(\frac{1}{x^3}\right) \Rightarrow$$
$$\Rightarrow x^a - ax^{a-1} + \frac{a(a-1)}{2}x^{a-2} + O\left(\frac{1}{x^3}\right)$$

Quindi è $\left(\frac{9}{2} a^3 (a-1) + \frac{a(a-1)}{2}\right) x^{a-2} =$

$= -\frac{2}{9} x^{-5/3}$