Alcuni esempi di semantica

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Astraiamo l'operatore Ternario

$$a ::= \underbrace{\ldots}_{\substack{\text{def.}\\ \text{originale}\\ \text{di } a}} \mid \underbrace{b}_{\substack{\text{cond.}\\ \text{booleana}}} ? \underbrace{a_0 : a_1}_{\substack{\text{expr.}\\ \text{aritm.}}}$$

(i)
$$\frac{\langle b, \sigma \rangle \to \text{true} \quad \langle a_0, \sigma \rangle \to n}{\langle b\,?\,a_0 : a_1, \sigma \rangle \to n}$$

(ii)
$$\frac{\langle b, \sigma \rangle \to \text{false} \quad \langle a_1, n \rangle \to n}{\langle b\,?\,a_0 : a_1, \sigma \rangle \to n}$$

Astraiamo $x{+}{+}$:

$$\frac{\langle x, \sigma \rangle \to n}{\langle x{+}{+}, \sigma \rangle \to \sigma' = \sigma[{}^{n+1}\!/x]}$$

COMANDI

$$\langle \text{skip}, \sigma \rangle \to \sigma$$

$$\frac{\langle a, \sigma \rangle \to m}{\langle x = a, \sigma \rangle \to \sigma[{}^{m}\!/a]}$$

Due expr. aritm. sono equivalenti quando $\forall \sigma \in \Sigma, \langle a_0, \sigma \rangle \to n \iff \langle a_1, \sigma \rangle \to n$ con $n \in \mathbb{N}$. Le regole si estendono anche ai comandi e ai booleani.

$\quad z = b ? a_0 : a_2 \equiv$ if b $z = a_0$ else $z = a_2$

(i) $$\frac{\langle b, \sigma \rangle \to \text{true} \quad \langle a_0, \sigma \rangle \to n}{\langle z = b ? a_0 : a_2, \sigma \rangle \to \sigma[^n/_z]} \Longleftrightarrow$$

$$\Longleftrightarrow \frac{\langle b, \sigma \rangle \to \text{true} \quad \langle a_0, \sigma \rangle \to n}{\langle \text{if } b \ z = a_0 \text{ else } z = a_2, \sigma \rangle \to \sigma[^n/_z]} \quad ✓$$

(ii) $$\frac{\langle b, \sigma \rangle \to \text{false} \quad \langle a_1, \sigma \rangle \to n}{\langle z = b ? a_0 : a_1, \sigma \rangle \to \sigma[^n/_z]} \Longleftrightarrow$$

$$\Longleftrightarrow \frac{\langle b, \sigma \rangle \to \text{false} \quad \langle a_2, \sigma \rangle \to n}{\langle \text{if } b \ z = a_0 \text{ else } z = a_2, \sigma \rangle \to \sigma[^n/_z]} \quad ✓$$

Def. ricorsiva dell'istruzione while

while b do $c \equiv$ if b then c; while b do c
$\qquad$ else skip

(i) $$\frac{\langle b, \sigma \rangle \to \text{true} \quad \langle c, \sigma \rangle \to \sigma'}{\langle \text{while } b \text{ do } c, \sigma \rangle \to \langle \text{while } b \text{ do } c, \sigma' \rangle}$$

(ii) $$\frac{\langle b, \sigma \rangle \to \text{false}}{\langle \text{while } b \text{ do } c, \sigma \rangle \to \sigma}$$