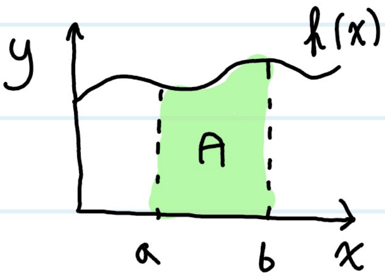
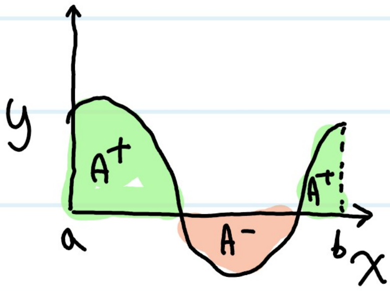
Integrali



estremi di integrazione

$$A = \int_a^b \underbrace{f(x)}_{\text{funzione integranda}}\,dx \qquad \text{(integrale indefinito)}$$



$$\int_a^b f(x)\,dx = A^+ - A^-$$

Inoltre $\displaystyle\int_b^a f(x)\,dx := -\int_a^b f(x)\,dx$

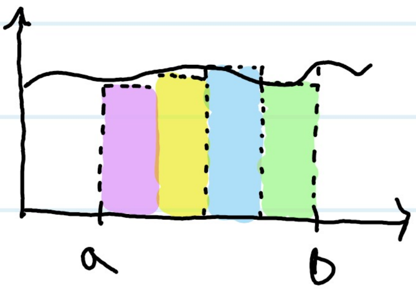
OSS.

$f \geq 0$ in $[a,b] \Longrightarrow \displaystyle\int_a^b f(x)\,dx \geq 0$

$f > 0$ in $\underset{a \neq b}{[a,b]} \Longrightarrow \displaystyle\int_a^b f(x)\,dx > 0$

OSS. 2

$$\int_a^a f(x)\,dx = 0$$



$$\int_a^b f(x)\,dx = \lim_{k\to+\infty} \sum_{i=0}^{k-1} \frac{b-a}{k} \cdot f\left(a+\frac{b-a}{k}\, i\right) = (*)$$

$$(*) = \boxed{\lim_{\delta\to 0} \sum_{i=1}^{N} f(R_i)\,\delta}$$

vero se f è continua

$$* \begin{cases} \delta = \frac{b-a}{k} \\ N = k \\ R_i = a + \delta(i-1) \end{cases}$$

Calcolo esatto dell'integrale

Si definisce primitiva di $f(x)$ una $F(x)$ t.c. $F'(x) = f(x)$.

Teorema $\left[\int f(x)\,dx\right]' = f(x)$

$$\lim_{h\to 0} \frac{\int_0^{x+h} f(t)\,dt - \int_0^{x} f(t)\,dt}{h} = \lim_{h\to 0} \frac{\int_x^{x+h} f(t)\,dt}{h} = (1)$$

$$\exists\, c \in [x, x+h] \mid f(c) = \frac{\int_x^{x+h} f(t)\,dt}{h}$$ (teorema del valor medio integrale)

$h \to 0 \Rightarrow c = x \Rightarrow$

$\Rightarrow (1) = f(x)$ $\square$