es. 1. 2017

(i)

		a	b
→	1	2	7
*	2	5	3
	3	6	5
*	4	1	5
	5	8	8
*	6	7	1
	7	8	4
*	8	3	7

$\{1,3,5,7\}$ $\{2,4,6,8\}$

a: $\{1,3,5,7\}$ $\{2,4,6,8\}$

b: $\{1,3\}$ $\{5,7\}$ $\{2,4,6,8\}$

$\{1,3\}$ $\{5,7\}$ $\{2,4,6,8\}$

a: $\{1,3\}$ $\{5,7\}$ $\{2,6\}$ $\{4,8\}$

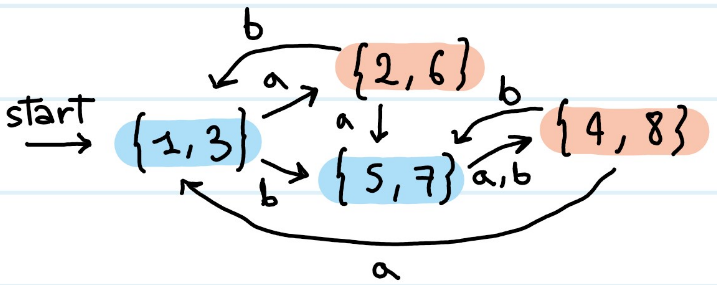
b: $\{1,3\}$ $\{5,7\}$ $\{2,6\}$ $\{4,8\}$

$\{1,3\}$ $\{5,7\}$ $\{2,6\}$ $\{4,8\}$ ✓

a: $\{1,3\}$ $\{5,7\}$ $\{2,6\}$ $\{4,8\}$

b: $\{1,3\}$ $\{5,7\}$ $\{2,6\}$ $\{4,8\}$

$\{1,3\}$ $\{5,7\}$ $\{2,6\}$ $\{4,8\}$



b

(ii) $A \cap B = (Q_A \times Q_B, \Sigma, \delta_{A \cap B}, (q_A, q_B), F_A \times F_B)$

con $\delta_{A\cap B} : ((q_1, q_2), a) \longmapsto (\delta_A(q_1, a), \delta_B(q_2, a))$.

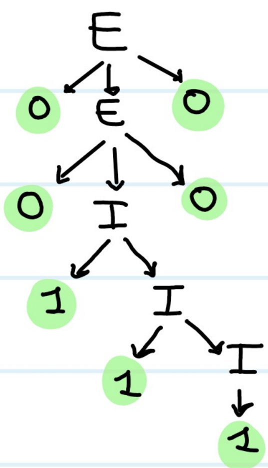
es. 2. 2017

$L_2 = L(G) = \{ 0^n 1^m 0^n \mid n > 0, m > 0 \}$

(i) $P = \{ E \to 0I0 \mid 0E0,\ I \to 1 \mid 1I \}$

$G = (\{E, I\}, \{0, 1\}, P, E)$ genera L_2.

(ii)



(iii) L_2 è libero perché generato dal linguaggio G.

Si assuma ora che L_2 sia regolare e abbia un suo DFA n stati. Per il Pumping lemma, $w = 0^n 1^n 0^n \in$

$\in L_2$ è t.c. $w = xyz \mid |xy| \leq n,\ y \neq \varepsilon,\ xy^iz \in L_2$ $\forall i \in \mathbb{N}$. Tuttavia y è composizione di soli 0, pertanto $xz \notin L_2$ perché non combacerebbe il numero di 0 da una parte all'altra, ↯. Quindi L_2 non è regolare.

es. 1.2015

(i) $P = \{E \to I \mid aEdd,\ I \to bc \mid bIc\}$
$G = (\{E, I\}, \{a, b, c, d\}, P, E)$ genera $L(G)$.

(ii) $P' = \{E \to aIdd \mid aEdd,\ I \to \varepsilon \mid bIc\}$
$G' = (\{E, I\}, \{a, b, c, d\}, P', E)$ genera $L(G')$.

es. 1.2014

(i) $L_0 = \{w \mid w \in \{0,1\}^*$ e w contiene almeno due 1 consecutivi$\}$

$P = \{E \to I11I,\ I \to \varepsilon \mid I0 \mid I1\}$
$G_0 = (\{E, I\}, \{0, 1\}, P, E)$ genera L_0.

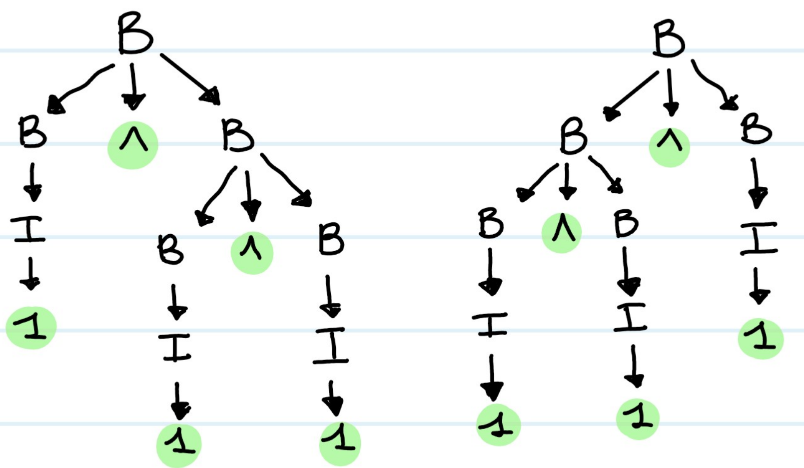
(ii) $L_1 = \{ w \mid w \in \{0,1\}^* \text{ e } w \text{ contiene più' } 1 \text{ che } 0 \}$

$$P = \{ E \rightarrow 1 \mid 1E0 \mid 0E1 \mid 10E \mid E10 \mid 01E \mid E01 \mid 1E \mid E1 \}$$

$$G = (\{E\}, \{0,1\}, P, E\}$$

(iii) $B \rightarrow I \mid B \wedge B \mid B \vee B \mid (B)$

$I \rightarrow 0 \mid 1$



poiché entrambi gli alberi sintattici hanno $1 \wedge 1 \wedge 1$ come forma sentenziale, si deduce che G_2 e' ambigua.

$$P = \{ B \rightarrow T \mid T \wedge B,\ T \rightarrow F \mid F \vee T,\ F \rightarrow 0 \mid 1 \mid (B) \}$$

$G = (\{B, T, F\}, \{0,1\}, P, B)$ e' equivalente a G_2 e non e' ambigua.

es. 1. 2012

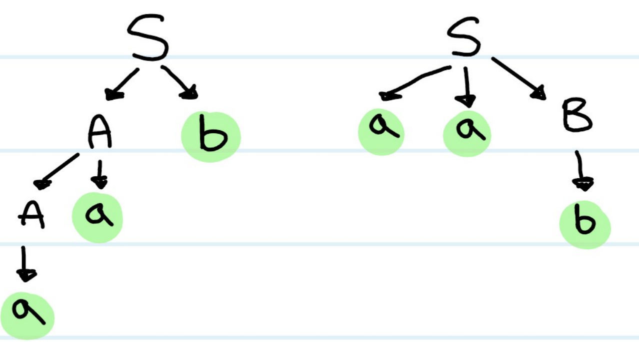
(i) $L/a = \{ w \in \Sigma^* \mid wa \in L \}$ con $a \in \Sigma$

Sia D un DFA che riconosce L, si costruisca il DFA D' che copi la struttura di D, ma che abbia come stati finali gli stati da cui mediante a si giunge a uno stato finale di D. D' accetta solamente L/a come linguaggio, quindi L/a è regolare.

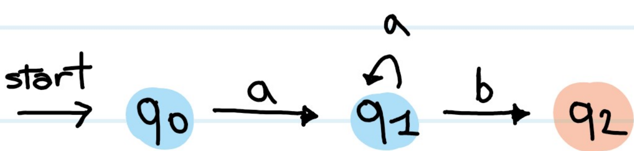
(ii) $S \to Ab \mid aaB$

$A \to a \mid Aa$

$B \to b$



Poiché entrambi gli alberi sintattici producono aab, la grammatica è ambigua.

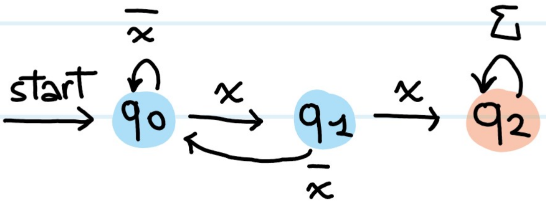


$P = \{ q_0 \to aq_1,\ q_1 \to aq_1 \mid bq_2,\ q_2 \to \varepsilon \}$

$G = (\{q_0, q_1, q_2\}, \{a, b\}, P, q_0)$

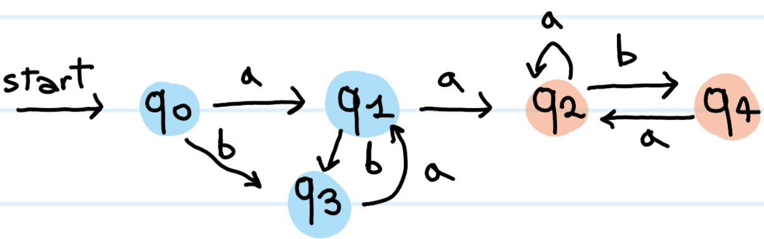
es. 1. 2011

$\Sigma = \{a, b\}$



$D(x)$ con $x \in \Sigma$

Il linguaggio L è intersezione di linguaggi regolari (i.e. $L(D(a)) \cap \cap \overline{L(D(b))}$), quindi è regolare.



accetta L.

$\{q_0, q_1, q_3\}$ $\{q_2, q_4\}$

a: $\{q_0, q_3\}$ $\{q_1\}$ $\{q_2, q_4\}$

b: $\{q_0, q_1, q_3\}$ $\{q_2\}$ $\{q_4\}$

$\{q_0, q_3\}$ $\{q_1\}$ $\{q_2\}$ $\{q_4\}$

a: $\{q_0, q_3\}$ $\{q_1\}$ $\{q_2\}$ $\{q_4\}$

b: $\{q_0\}$ $\{q_1\}$ $\{q_2\}$ $\{q_3\}$ $\{q_4\}$

$\{q_0\}$ $\{q_1\}$ $\{q_2\}$ $\{q_3\}$ $\{q_4\}$ ✓

(l'automa era già minimo)

es. 1. 2010

(i)

	0	1
$\rightarrow q_0$	q_1	q_2
q_1	q_0	q_3
q_2	q_4	q_3
q_3	q_1	q_5
$* q_4$	q_4	q_5
q_5	q_4	q_3

$\{q_0, q_1, q_2, q_3, q_5\}$ $\{q_4\}$

0: $\{q_0, q_1, q_3\}$ $\{q_2, q_5\}$ $\{q_4\}$

1: $\{q_0, q_1, q_2, q_3, q_5\}$ $\{q_4\}$

$\{q_0, q_1, q_3\}$ $\{q_2, q_5\}$ $\{q_4\}$

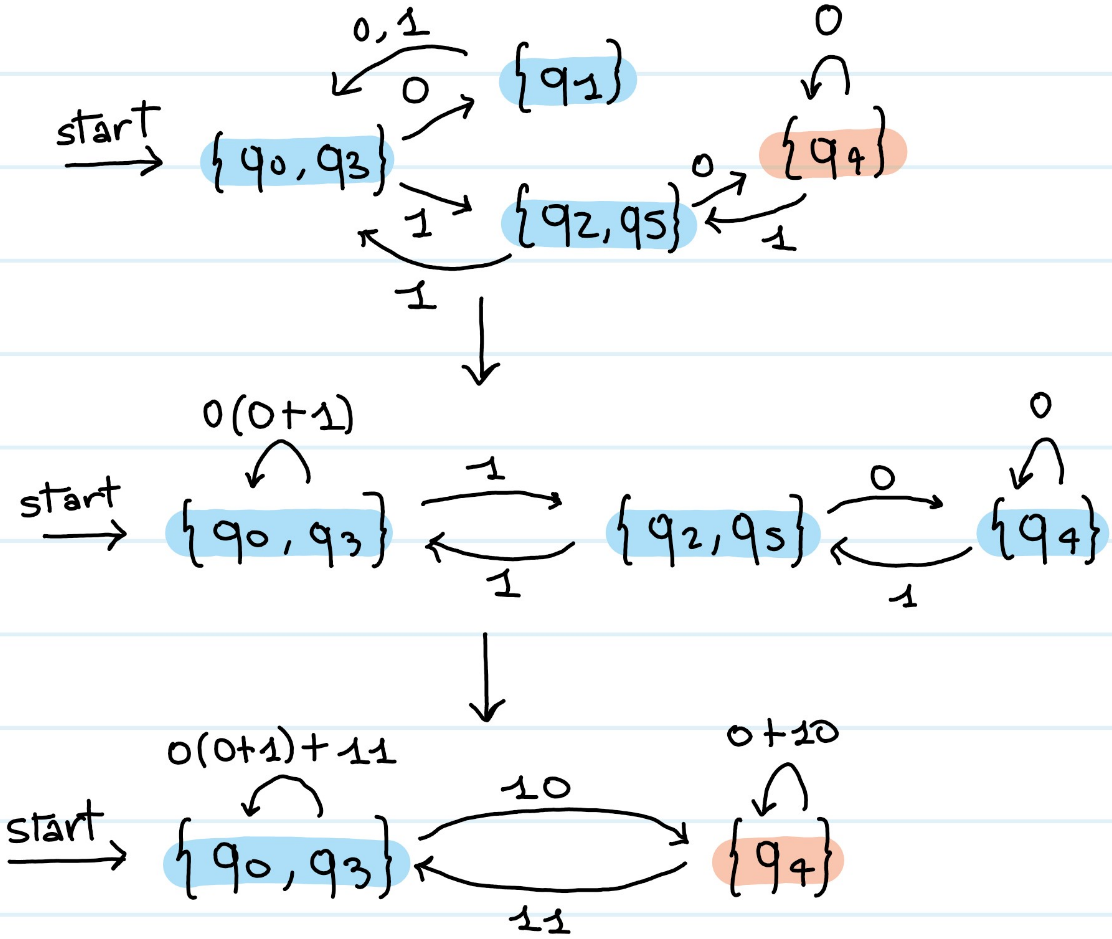
0: $\{q_0, q_1, q_3\}$ $\{q_2, q_5\}$ $\{q_4\}$

1: $\{q_0, q_3\}$ $\{q_1\}$ $\{q_2, q_5\}$ $\{q_4\}$

$\{q_0, q_3\}$ $\{q_1\}$ $\{q_2, q_5\}$ $\{q_4\}$ ✓

0: $\{q_0, q_3\}$ $\{q_1\}$ $\{q_2, q_5\}$ $\{q_4\}$

1: $\{q_0, q_3\}$ $\{q_1\}$ $\{q_2, q_5\}$ $\{q_4\}$



$$(0(0+1)+11+10(0+10)^*11)^*10(0+10)^*$$

(ii) $P = \{ \{q_0,q_3\} \to 0\{q_1\} \mid 1\{q_2,q_5\},\ \{q_1\} \to 0\{q_0,q_3\} \mid 1\{q_0,q_3\},\ \{q_2,q_5\} \to 0\{q_4\} \mid 1\{q_0,q_3\},\ \{q_4\} \to 0\{q_4\} \mid 1\{q_2,q_5\} \mid \varepsilon \}$

$G = (\{ \{q_0,q_3\}, \{q_1\}, \{q_2,q_5\}, \{q_4\} \}, \{0,1\}, P,$

$\{q_0, q_3\})$.

es. 1. 2009

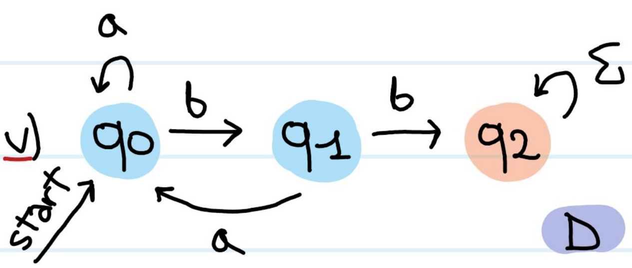
(i) $(a+b)^* bb (a+b)^*$

(ii) $P = \{ E \rightarrow IbbI,\ I \rightarrow \varepsilon \mid aI \mid bI \}$

$G = (\{E, I\}, \{a, b\}, P, E)$ genera L.

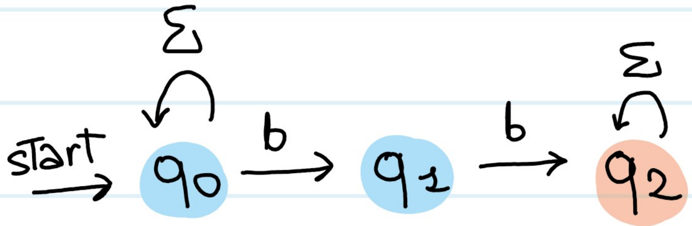
(iii) $E \Rightarrow IbbI \Rightarrow aIbbI \Rightarrow abbI \Rightarrow abbbI \Rightarrow$

$\Rightarrow abbbbI \Rightarrow abbbbaI \Rightarrow abbbba$

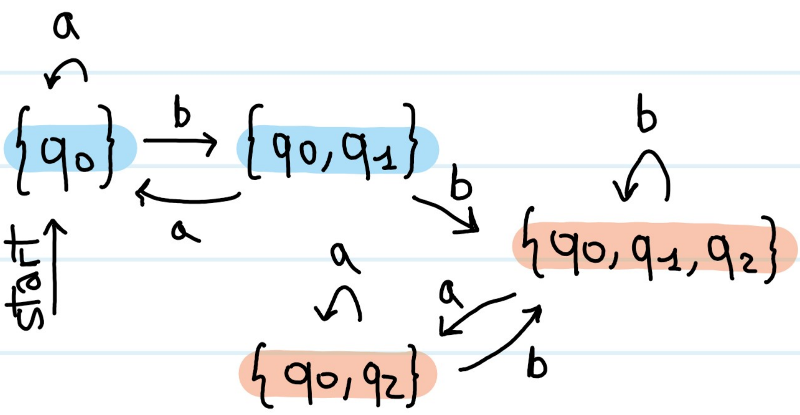
(iv e v) 

(un DFA è un NFA)

altrimenti



	a	b
$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_1\}$	$\emptyset$	$\{q_2\}$
$\{q_2\}$	$\{q_2\}$	$\{q_2\}$
$\{q_0, q_1\}$	$\{q_0\}$	$\{q_0, q_1, q_2\}$



$\{q_0,q_2\}$ $\{q_0,q_2\}$ $\{q_0,q_1,q_2\}$

$\{q_0,q_1,q_2\}$ $\{q_0,q_2\}$ $\{q_0,q_1,q_2\}$

$\{\{q_0\},\{q_0,q_1\}\}$ $\{\{q_0,q_1,q_2\},\{q_0,q_2\}\}$

a: $\{\{q_0\},\{q_0,q_1\}\}$ $\{\{q_0,q_1,q_2\},\{q_0,q_2\}\}$

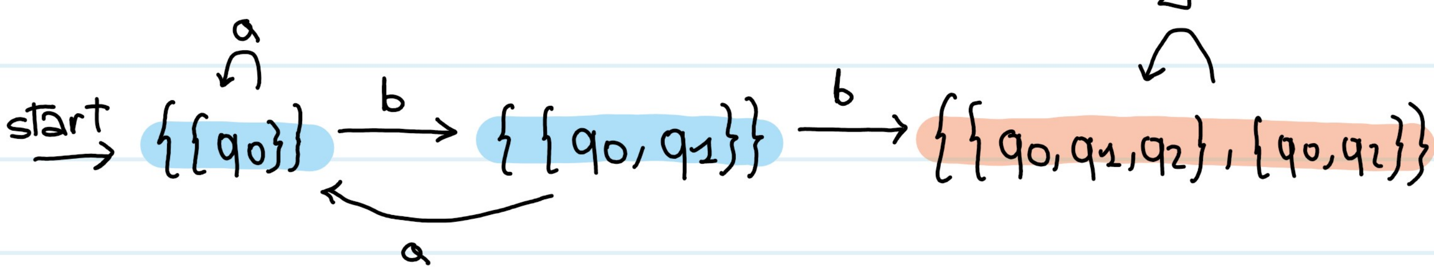
b: $\{\{q_0\}\}$ $\{\{q_0,q_1\}\}$ $\{\{q_0,q_1,q_2\},\{q_0,q_2\}\}$

$\{\{q_0\}\}$ $\{\{q_0,q_1\}\}$ $\{\{q_0,q_1,q_2\},\{q_0,q_2\}\}$ ✓

a: $\{\{q_0\}\}$ $\{\{q_0,q_1\}\}$ $\{\{q_0,q_1,q_2\},\{q_0,q_2\}\}$

b: $\{\{q_0\}\}$ $\{\{q_0,q_1\}\}$ $\{\{q_0,q_1,q_2\},\{q_0,q_2\}\}$

Quindi l'NFA è equivalente a:



Ossia il DFA inizialmente presentato (i.e. D), che così si dimostra essere anche minimo.

(vi) (in riferimento a D)

$P = \{ q_0 \to a q_0 \mid b q_1,\ q_1 \to a q_0 \mid b q_2,\ q_2 \to a q_2 \mid b q_2 \mid \varepsilon \}$

$G = (\{ q_0, q_1, q_2 \}, \{ a, b \}, P, q_0)$