es. $e^x \geq \frac{5}{2}x \quad \forall x \in \mathbb{R}$?

$$e^x - \frac{5}{2}x \geq 0 \Rightarrow \min \underbrace{e^x - \frac{5}{2}x}_{g(x)} \geq 0$$

$$g'(x) = 0 \Rightarrow x = \ln\left(\frac{5}{2}\right)$$

$$M = \left\{\pm\infty, \frac{5}{2}\right\}$$

$$g(x) \xrightarrow[x \to +\infty]{} +\infty \qquad g(x) \xrightarrow[x \to -\infty]{} +\infty$$

$$\min e^x - \frac{5}{2}x = \frac{5}{2} - \frac{5}{2}\ln\left(\frac{5}{2}\right) =$$

$$= \frac{5}{2}\left(\underbrace{1 - \ln\left(\frac{5}{2}\right)}_{>0}\right) > 0 \quad ✓$$

Oppure: $e^x = ax$

$$y = \underbrace{e^{x_0}}_{a}(x - x_0) + e^{x_0} =$$

$$= \overbrace{e^{x_0}}^{a} x + e^{x_0}(1 - x_0)$$

$$\underbrace{e^{x_0}}_{a}(1 - x_0) = 0 \Rightarrow$$

$$\Rightarrow a(1 - \ln(a)) = 0 \Rightarrow$$

$$\Rightarrow a = e \qquad \boxed{a \geq e}$$

es. $a, b \mid \quad a(1+x^4) \leq (1+x)^4 \leq b(1+x^4) \quad \forall x \in \mathbb{R}$

$$\Longrightarrow \quad a \leq \frac{(1+x)^4}{(1+x^4)} \leq b$$

$$g(x) \xrightarrow[x \to \pm\infty]{} +\infty$$

$$\min \underbrace{\frac{(1+x)^4}{(1+x^4)}}_{g(x)} = (1)$$

$$g'(x) = \frac{4(1+x)^3(1+x^4) - 4x^3(1+x)^4}{(1+x^4)}$$

$$\Updownarrow \quad (g'(x) = 0)$$

$$x = \pm 1$$

$$(1) = 0 \quad \Longrightarrow \quad a \leq 0$$

$$\max \frac{(1+x)^4}{(1+x^4)} = 8 \quad \Longrightarrow \quad b \geq 8$$

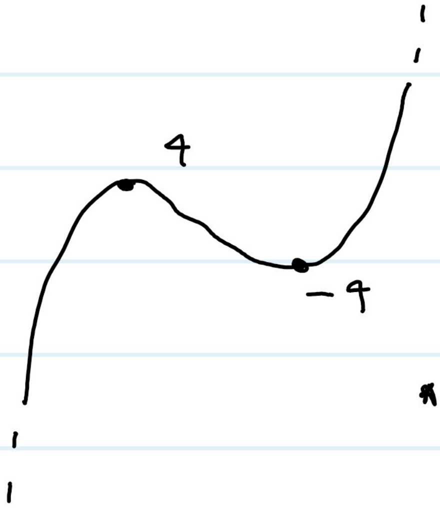
es. $\forall a \in \mathbb{R}$ quante soluzioni ha $\underbrace{x^5 - 5x}_{h(x)} = a$?

$$f'(x) = 5x^4 - 5 = 5(x^4 - 1)$$

$$f'(x) = 0 \quad \Longrightarrow \quad x = \pm 1$$

$$\nearrow \quad -1 \quad \searrow \quad 1 \quad \nearrow$$

$f(-1) = 4 \qquad f(1) = -4$



$a > 4 \vee a < 4$: 1 soluzione

$a = \pm 4$: 2 soluzioni *

$-4 < a < 4$: 3 soluzioni

* con 1 o -1 contato con doppia molteplicità.