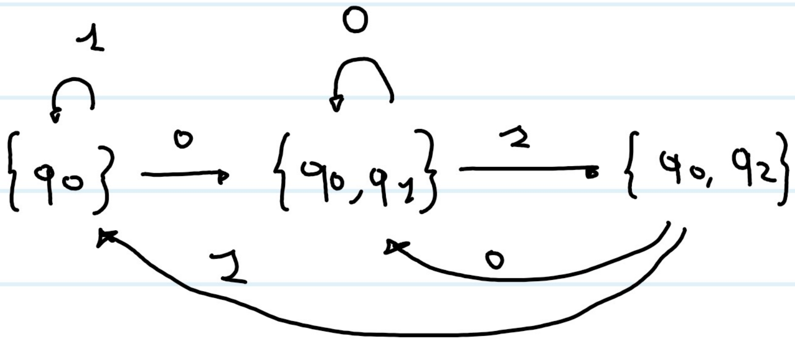
Equivalenza tra DFA e NFA

E' sempre possibile trovare un DFA equivalente a un NFA, ossia che accetti il suo stesso linguaggio.

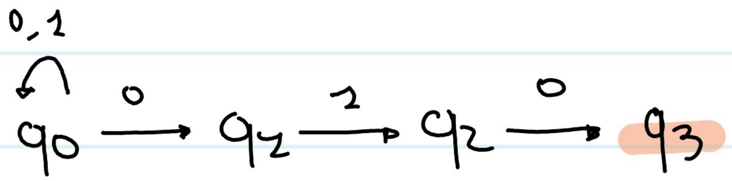
Dato l'NFA $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ costruiremo $D = (Q_D, \Sigma, \delta_D, q_0, F_D)$.

- $Q_D = \{ S \mid S \subseteq Q_N \}$ $\left(|Q_D| = 2^{|Q_N|} \right)$
- $F_D = \{ S \subseteq Q_N \mid S \cap F \neq \emptyset \}$
- $\delta_D(S, a) = \bigcup_{p \in S} \delta_N(p, a)$

es.

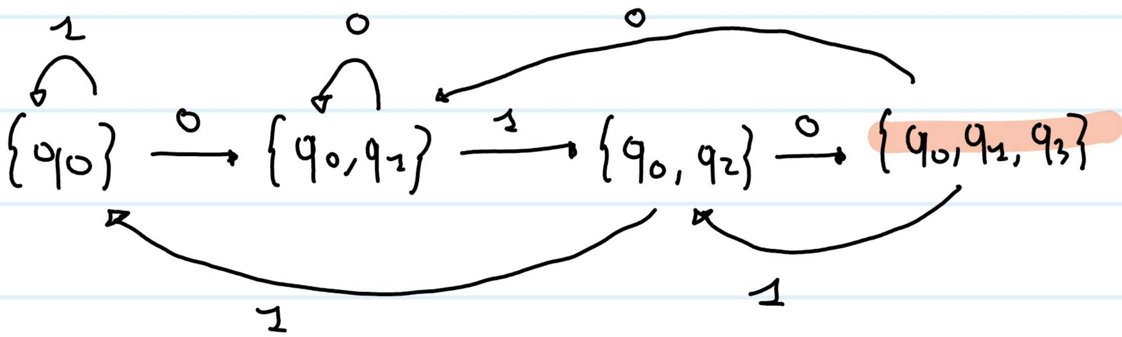


es.



	0	1
$\to q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\emptyset$	$\{q_2\}$
q_2	$\{q_3\}$	$\emptyset$
q_3	$\emptyset$	$\emptyset$

	0	1
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_1\}$	$\emptyset$	$\{q_2\}$
$\{q_2\}$	$\{q_3\}$	$\emptyset$
$\{q_3\}$	$\emptyset$	$\emptyset$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



Teorema $L(D) = L(N)$

Mostriamo che $\hat{\delta}_D(\{q_0\}, w) = \hat{\delta}_N(q_0, w)$

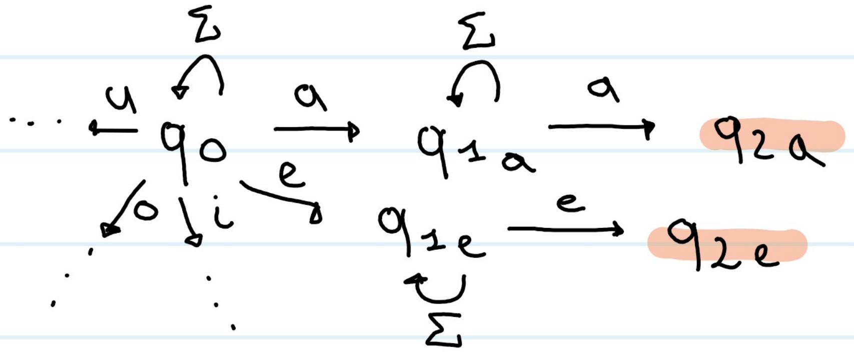
- **base:** $\hat{\delta}_D(\{q_0\}, \varepsilon) = \hat{\delta}_N(q_0, \varepsilon)$ ✓
- **induzione:** $\hat{\delta}_D(\{q_0\}, xa) = \bigcup_{p \in \hat{\delta}_D(\{q_0\}, x)} \hat{\delta}_N(p, a) =$

$$= \bigcup_{p \in \hat{\delta}_N(q_0, x)} \hat{\delta}_N(p, a) = \hat{\delta}_N(q_0, xa)$$ ✓

Quindi i due automi mandano stesse stringhe in stessi stati, inclusi quelli finali. Pertanto $L(D) = L(N)$. $\boxtimes$

es. $\Sigma = \{a, e, i, o, u\}$

- vocale finale già apparsa



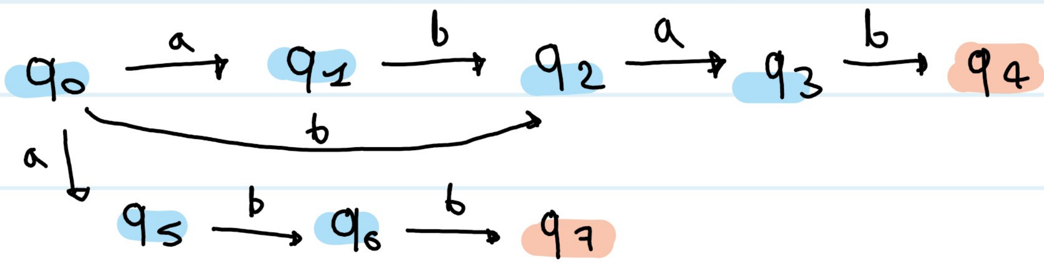
aeea

eee

es. $\Sigma = \{a, e, i, o, u\}$

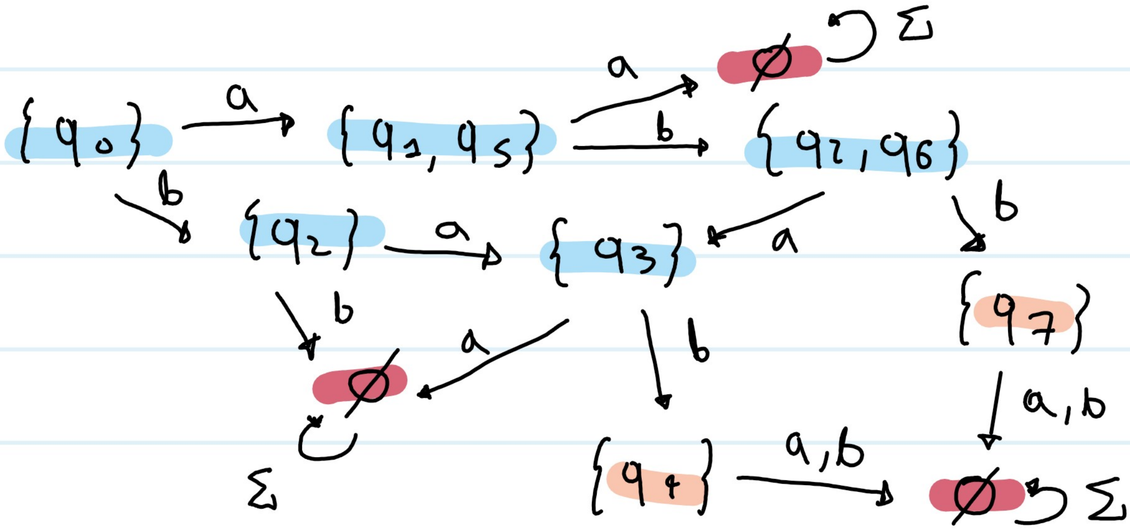
- vocale finale NON apparsa.

es. $\Sigma = \{a, b\}$ abab, bab, abb



	a	b
$\rightarrow \{q_0\}$	$\{q_1, q_5\}$	$\{q_2\}$
$\{q_1\}$	$\emptyset$	$\{q_2\}$
$\{q_2\}$	$\{q_3\}$	$\emptyset$
$\{q_3\}$	$\emptyset$	$\{q_4\}$
$* \{q_4\}$	$\emptyset$	$\emptyset$
$\{q_5\}$	$\emptyset$	$\{q_6\}$
$\{q_6\}$	$\emptyset$	$\{q_7\}$

$*\{q_7\}$	$\emptyset$	$\emptyset$
$\{q_1, q_5\}$	$\emptyset$	$\{q_2, q_6\}$
$\{q_2, q_6\}$	$\{q_3\}$	$\{q_7\}$



es. $\Sigma = \{0, 1\}$ $\quad x\,011\,y$

$$\overset{\Sigma}{\circlearrowleft} q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{1} q_3 \circlearrowright \Sigma$$

- mostriamo che accetta $x011$:
  - $q_0 \in \hat{\delta}(q_0, w) \ \forall w \in \Sigma^*$
  - $\hat{\delta}(q_0, x011) = \bigcup_{p \in \hat{\delta}(q_0, x)} \hat{\delta}(p, 011) \supseteq$
    $\supseteq \hat{\delta}(q_0, 011) \ni q_3 \Rightarrow$
    $\Rightarrow$ è uno stato finale.
- mostriamo che accetta $x011y$:

- $q_3 \in \hat{\delta}(q_3, w) \ \forall w \in \Sigma^*$
- $\hat{\delta}(q_0, x011y) = \bigcup_{p \in \delta(q_0, x011)} \hat{\delta}(p, y) \supseteq$

$\supseteq \hat{\delta}(q_3, y) \ni q_3 \rightarrow$

$\Rightarrow$ è uno stato finale. $\boxtimes$